I would like to solve the following boundary-value problem that originates from a simplified version of Iverson’s theory, which was presented in the paper “Steady and intermittent slipping in a model of landslide motion regulated by pore-pressure feedback” by D.G. Schaeffer and R.M. Iverson (SIAM J. Appl. Math. 69 769–786 2008). Iverson’s model involves a linear diffusion equation and a first-order differential equation:

\[
\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial y^2} \quad \text{for } 0 < y < 1, \tag{1}
\]

\[
p(0,t) = \psi v(t), \tag{2}
\]

\[
p(1,t) = 0, \tag{3}
\]

where \( p \) is the pore pressure, \( \psi \) a constant, and \( v \) (definite positive) is the velocity given by the following ODE coupled with the basal pore pressure

\[
\frac{\mathrm{d}v}{\mathrm{d}t} = \sin \theta - \mu(v)(A_1 \cos \theta - A_2 p(0,t)), \tag{4}
\]

with \( \theta \) the bottom slope, \( A_1, A_2, \) and \( \epsilon \) constant parameters. I also introduced the friction function

\[
\mu(v) = \mu_0 \left( 1 - a \sinh \left( \frac{K}{2v_{ref}} v \right) \right), \tag{5}
\]

where \( \mu_0, a, v_{ref}, K \) denote constitutive parameters, which are constant. These governing equations for \( p(y,t) \) and \( v(t) \) are also subject to initial conditions in the form \( p(y,0) = p_0 \) and \( v(0) = v_0 \). Note that since \( v \geq 0 \), (4) holds only when the block is sliding.

To solve the equations (1)–(4), I used a fractional-step approach:

- I solved (4) to compute an intermediate velocity \( v^{k+1} \), e.g.

  \[
  v^{k+1} = v^k + dt F(v^k, p^k),
  \]

  where \( v^k \) denotes the velocity at time \( t_k \) and \( p = (p_1, \cdots, p_n) \) is the discretized pressure vector.

- They I solved (1) using a Crank-Nicolson scheme, e.g.,

  \[
  A \cdot p^{k+1} = B \cdot p^k + C(v^k, v^{k+1}),
  \]

  with \( A \) and \( B \) the usual Crank-Nicolson matrices and \( C \) a vector column that accounts for the boundary condition at \( y = 0 \)

  \[
  C = \begin{pmatrix}
  -2\psi \frac{\mathrm{d}t}{\mathrm{d}y} v^{k+1} + v^k \\
  \vdots \\
  0
  \end{pmatrix}.
  \]