Numerical Calculation of the Dam-Break Riemann Problem with a Detailed Method and Comparison with a Simplified Method

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Lausanne, im November 2008
Task

In recent years, many dams of small size have been built in the Alps to supply ski resorts with water or to produce artificial snow. Contrary to large dams, subject to international design rules, small dams are exempt from specific standards that enforce their safety. To prevent disasters, local authorities in Switzerland (and in other countries) try to impose higher safety standards, in particular as concerns dam stability and dam-break induced floods. The Federal Office for Water and Geology (Bundesamt für Wasser und Geologie) published several recommendations and methods related to dam safety [1]. Since hydraulics on steep slope remains a difficult topic, emphasis was given to simple assessment methods. At the same time, however, a number of professionals questioned the reliability of such methods (e.g., see [2]) since they overly simplify the physical processes at play (including intense sediment transport, proper evaluation of the surge velocities, etc.). Another point of concern is that most numerical models used by practitioners are devoted to shallow slopes. The governing equations are usually not appropriate to computing flows on steep slope and the numerical methods may fail because of numerical instabilities.

The objective of the thesis is to compare simplified and detailed computation methods. The student will use a Fortran library called Clawpack (developed by Prof. Randall LeVeque, University of Washington, USA) to obtain high accuracy numerical solutions. She then will compare the outcomes with the results provided by simple methods. Emphasis will be first put on clearwater flows (that is, with no sediment transport). Then, attention should be focused on the effects of intense sediment transport induced by a dam-break wave. Real case studies will be carried out to exemplify the differences between both approaches.

The student will be supervised by Prof. Christophe Ancey and his PhD Student, Martin Rentschler.

Abstract

This work presents theoretical foundations of the numerical computation of dam-break flood waves based on the Godunov method. Furthermore, bed-load transport models and characteristics of debris-flows, occurring in case of steep slopes, are elucidated as well as simplified methods for flood wave computation which are recommended by federal offices are presented.

An extension of the Fortran-library CLAWPACK is elaborated, allowing for simulation of the one-dimensional dam-break flood wave over irregular beds. Consideration of viscosity, friction and sediment transport is enabled. The written program is validated on basis of analytical solutions and experimental results for dam-breaks. Good agreement of the quantities water height, velocity and discharge is found; results are locally influenced by numerical diffusion.

A comparison between results obtained with CASTOR, a program basing on a simplified method and recommended by federal offices, is drawn. The outcomes of the elaborated program are considered as reference solutions. Test cases for small reservoir volumes and different bed slopes are carried out. Considerable differences in the results for water height, velocity and discharge are found. The simplified computation yields unrealistic graphs for these quantities and it is doubtful if application to cases such as the considered ones is recommendable and if decisions influencing safety of the general public should be taken basing on results which the method provides.

The elaborated program is applied to simulate a laboratory experiment of sediment transport under dam-break flood waves. Hydraulic parameters are computed in agreement with observed data. The shape of the sediment displacements agrees as well but sediment transport rates calculated with the applied bed-load transport model are considerably too small. Currently available sediment transport models have been developed for application in rivers. As dam-breaks cause significantly differing flow conditions, the considered transport formulas can be regarded as not applicable to these cases.
Zusammenfassung


Das erstellte Programm wird zur Simulation eines Laborexperiments zum Sedimenttransport bei Dammbrüchen verwendet. Hydraulische Parameter werden in Übereinstimmung mit experimentellen Ergebnissen ermittelt. Die Form der Bodenverlagerungen wird ebenfalls vergleichbar berechnet, das verwendete Transportmodell führt jedoch zu deutlich zu geringen Transportraten. Derzeit zur Verfügung stehende Geschiebetransportformeln sind für die Anwendung in Flüssen entwickelt worden. Da bei Dammbrüchen signifikant abweichende Strömungsverhältnisse herrschen, sind die betrachteten Transportformeln als auf diese Fälle nicht übertragbar zu bezeichnen.
Preface

This thesis was written at the Environmental Hydraulics Laboratory (LHE) of the École Polytechnique Fédérale of Lausanne (EPFL) in the autumn of 2008. It was supervised by Prof. Christophe Ancey and his PhD Student, Martin Rentschler.

Je remercie Monsieur Christophe Ancey de m’avoir donné la possibilité d’écrire ce mémoire dans son groupe de recherche et d’avoir assumé la tâche d’examineur.

Herrn Martin Rentschler danke ich für die wissenschaftliche Betreuung der Arbeit und anregenden Diskussionen während der Entstehung.

Bei Herrn Werner Zielke vom Institut für Strömungsmechanik und Elektronisches Rechnen im Bauwesen (ISEB) der Leibniz Universität Hannover möchte ich mich dafür bedanken, dass er die Aufgabe des Erstprüfers übernommen und mich bei der Anmeldung der Arbeit an meiner Heimuniversität unterstützt hat.

In diesem Zusammenhang sei auch Herrn Rainer Ratke gedankt, der ebenfalls als Prüfer fungiert.


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Notation: Latin Letters

$A^\pm \Delta \Phi$ change in $\Phi$ through left($-$)- or right($+$)-going waves
$a_{1,2,3}$ geometrical parameters of the force equilibrium at a particle
$b_{1,2,3}$ geometrical parameters of the force equilibrium at a particle
$C$ overall Chézy coefficient
$C'$ grain-related Chézy coefficient
$C_D$ drag coefficient
$CFL$ Courant- or Courant-Friedrichs-Lewy number
$c$ wave velocity
$c_b$ bed-load concentration
$D_s$ dimensionless particle diameter
$d$ (particle) diameter
$e_{ij}, \mathbf{E}$ deformation tensor
$\mathbf{F}$ force
$\mathbf{F}_D$ drag force
$\mathbf{F}_L$ lift force
$Fr$ Froude number
$f(\phi)$ flux function vector
$G$ weight force
$g$ gravity
$g(\phi)$ flux function vector
$h$ water height
$\dot{I}$ momentum
$I$ unity tensor
$J$ slope
$\ddot{k}$ body force
$k_{St}$ Strickler coefficient
$k_\beta$ Schoklitsch correction factor
$k_\gamma$ Leiner correction factor
**Notation**

$L$  
> typical length scale for the computation of the Reynolds number

$M$  
> set of material points of a body

$m$  
> mass

$\vec{n}$  
> normal vector

$p$  
> pressure

$q_s$  
> volumetric bed-load transport rate

$r$  
> eigenvector

$S$  
> surface bounding $V$

$s$  
> Rankine-Hugoniot condition satisfying speed

$s$  
> ratio of sediment density and water density: $s = \rho_s/\rho$

$Re$  
> Reynolds number

$Re_*$  
> grain related Reynolds number, $Re_* = u_*d/\nu$

$T$  
> dimensionless bed shear parameter

$t$  
> time

$\vec{t}$  
> stress or traction vector

$U$  
> typical velocity for the computation of the Reynolds number

$u_b$  
> particle velocity

$u_*$  
> shear velocity

$\vec{u}$  
> velocity vector

$u, v, w$  
> its components

$V$  
> region in space

$\mathcal{V}$  
> control volume

$W$  
> wave

$\vec{x}$  
> vector of a position

$Z$  
> evaluation of $z(\phi)$ for $\Phi$

$z(\phi)$  
> parameter vector of $\phi$
Greek Letters

\[ \alpha \] scalar (indicates a scalar multiple of the adjacent quantity)
\[ \gamma \] limiter function
\[ \Delta x \] grid width
\[ \delta_b \] saltation height
\[ \delta_{ij} \] Kronecker delta
\[ \eta \] dynamic viscosity
\[ \theta \] mobility parameter
\[ \theta_{cr} \] Shields parameter
\[ \kappa \] local smoothness in a wave field
\[ \lambda \] eigenvalue
\[ \lambda^* \] second viscosity
\[ \mu \] bed-form or efficiency factor (ratio of Chézy coefficients)
\[ \nu \] kinematic viscosity
\[ \xi \] ratio \( x/t \)
\[ \rho \] water density
\[ \rho_s \] sediment density
\[ \tau, \mathbf{T} \] stress tensor
\[ \tau_b \] bed shear stress
\[ \tau_{cr} \] critical bed shear stress (begin of particle motion)
\[ \Phi \] conserved state variable \( \phi \), averaged in a cell
\[ \phi \] in vector equation notation: conserved state variable, first order tensor or scalar
\[ \bar{\phi} \] cell average function
\[ \tilde{\phi} \] integral curve
\[ \varphi \] source term vector
\[ \Psi \] continuous scalar or tensor function of any order
\[ \Omega \] two-dimensional projection of the control volume onto the \( \{x,y\}\)-plane
\[ \partial \Omega \] surface of \( \Omega \)
Sub- and Superscripts

\[ c \] \quad \text{state of a variable along a contact discontinuity}
\[ i,j,k \] \quad \text{quantity in the } i,j,k \text{th coordinate direction}
\[ l \] \quad \text{left state of a variable}
\[ m \] \quad \text{middle or intermediate state of a variable, connected to adjacent states } l \text{ and } r \text{.}
\[ n \] \quad \text{time step number}
\[ p \] \quad \text{p} \text{th entry of a vector}
\[ r \] \quad \text{right state of a variable}
\[ \text{red} \] \quad \text{due to friction effects reduced variable}
\[ t \] \quad \text{derivation with respect to the time}
\[ x \] \quad \text{derivation with respect to the x-direction}
\[ y \] \quad \text{derivation with respect to the y-direction}
\[ 0 \] \quad \text{initial state of a variable}
\[ * \] \quad \text{fixed left or right state}
\[ i \] \quad \text{cell number}
\[ 1 \] \quad \text{first entry of a vector}
\[ 2 \] \quad \text{second entry of a vector}
\[ + \] \quad \text{dimensionless quantity OR positive value}
\[ - \] \quad \text{negative value}
\[ ′ \] \quad \text{differentiated quantity, e.g. in case of the Jacobian matrix}
\[ \hat{\cdot} \] \quad \text{approximated state of a quantity OR quantity according to the Roe-linearization}
1. Introduction

Man use the principle of creating water reservoirs through dams and similar constructions for many centuries now, intensified since the invention of the water wheel in the XVIth century. In these times, the impounded water ensured the operation of e.g. flour, hammer and cutting mills. Today, most dam plants are build with the aim to provide electricity or drinking water. Hereby characteristic dimensions such as those of the Gepatsch dam in Austria are 150 mio m$^3$ for the dammed water volume and 600 m and 130 m for dam length and height, respectively. In constructions of these orders of magnitude a high risk potential for life and commodity values is inherent: When in 1943 the Moehne dam breached$^1$, 1600 people died. In order to prevent those losses in preparing emergency action plans or in exempting certain regions from population, it is of public concern to obtain knowledge of hydraulically relevant parameters of the occurring flood wave.

Being aware of the quantities water height, velocity and discharge is not only for failure cases of dams of the above-mentioned dimensions of relevance, but also for the considerably smaller ones which are present in alpine skiing areas. These serve as reservoirs for production of artificial snow and are characterised by capacities of about 1500 m$^3$. Even though this volume seems marginal in comparison to the more widespread dams of drinking water and energy supply, an under no circumstances negligible danger emanates from smaller dams. Whereas for a long time official regulations and research groups took these constructions in an unsatisfactory way into account, in Switzerland efforts are being made to enlarge safety requirements imposed on this type of dams in dependence on the possibly occurring flood wave.

To ensure common safety, the physical reliability of computation of dam-break flood waves is thus of exceeding importance. But even if simplifying breaching scenarios are assumed this task is challenging: Due to steep slopes and the high potential energy of the stored water unsteady and non-uniform flow-conditions are present which complicate calculation and can almost only be resolved by numerical methods. In addition, processes like sediment transport are to be taken into account.

Subject of this thesis is the computation of the flood wave in case of instantaneous, complete failure of the dam. In adding subroutines to the Fortran-library CLAWPACK$^2$, a program was written which simulates an one-dimensional wave on arbitrary bed geometries based on the shallow water equations. Effects of viscosity and friction are accounted for as well as computation of bed-load transport and bed-evaluation is possible. Objective of elaboration of the program is a comparison between results obtained with it as reference solution and computations basing on simplified methods applied in federal offices.

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$^1$The Moehne reservoir, with a volume of 140 mio m$^3$, 600 m dam length and 130 m dam height, was partly by British bombers, attacking it in the night of 16-17 May 1943 with newly constructed bouncing bombs.

$^2$Developed at the University of Washington, [1]
The present work presents the relevant theoretical foundations, results of the validation of the program and a comparison between flood wave results determined with the elaborated program and with another one, recommended by Swiss federal offices.

Section 2 shows the derivation of the shallow water equations beginning with the Navier-Stokes equations. Section 3 considers the problem of discontinuities by utilization of the shallow water equations in their differential form and the solution of the weak form. Subject of section 4 are the characteristic structure of a dam-break and the Riemann solution. Numerical methods in form of the applied Godunov method, the Roe linearization and the HLL- and HLLE-solver are presented in section 5, as well as high-resolution methods and numerical peculiarities in dam-break flood wave computation. Section 6 contains theoretical foundations of sediment transport and descriptions of transport models for different bed slopes and of debris-flows which occur in steep regions as the Alps.

The extension of the library CLAWPACK for elaboration of the program is presented in section 7 and validated in section 8, where results of the simulation of different theoretical and experimental test cases are given. Content of section 9 is the comparison of results obtained with CLAWPACK and a simplified method. A simulation of sediment transport and applicability of transport models to dam-break cases are discussed in section 6.

Section 11 points out possibilities of future works; the closing section 12 sums up obtained findings.

Einleitung


\[ \text{Die Möhnetalsperre, eingeweiht 1913 und über ein Stauziel von 140 Mio m}^3 \text{ verfügend bei einer Mauerlänge von 650 m und einer -höhe von 40 m, wurde in der Nacht zum 17. Mai 1943 durch britische Bomber mithilfe zu diesem Zweck entwickelter Rollbomben teilweise zerstört.} \]
Schnee und verfügen über Fassungsvermögen um 5000 m³. Auch wenn diese Volumina im Vergleich zu denen der häufiger zu findenden, für die allgemeine Wasser- und Energieverorgung dienenden Talsperren gering sind, geht auch von diesen Reservoirn eine unter keinen Umständen zu vernachlässigende potentielle Gefahr aus. Wurden diese Bauwerke von behördlichen Bestimmungen und Forschungsgruppen bisher nur unbefriedigend berücksichtigt, bemüht man sich in der Schweiz derzeit darum, in Abhängigkeit von der möglicherweise eintretenden Flutwelle höhere Sicherheitsanforderungen an ihre Bemessung zu stellen.


4Entwickelt an der University of Washington, [1]
2. Derivation of the Shallow Water Equation

An approximation of common practice for the calculation of dam-break floods is the utilization of the shallow water equations. This section presents their derivation out of the Navier-Stokes equations. The Navier-Stokes equations are derived in appendix A.

2.1. Euler’s Equation

The Navier-Stokes equation reads
\[
\rho \frac{D u_i}{D t} = \rho k_i + \frac{\partial}{\partial x_i} \left\{ -p + \lambda^* \frac{\partial u_k}{\partial x_k} \right\} + \frac{\partial}{\partial x_j} \left\{ \eta \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \right\},
\]
(2.1)

with the material derivative
\[
\frac{D}{D t} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i}
\]
(2.2)

the body force vector \( \vec{k} \), the pressure \( p \), the second viscosity \( \lambda^* \) and the dynamic viscosity \( \eta \). A derivation of this equation can be found in appendix A.

The Euler equation governs the dynamics of a compressible material, e.g. gases or liquids at high pressures, and is usually used for the description of flow in gas turbines. It neglects the effects of body forces, viscous stresses and heat conduction, so the Navier-Stokes equation (2.1) yields
\[
\rho \frac{D u_i}{D t} = \rho k_i - \frac{\partial p}{\partial x_i}
\]
(2.3)
or
\[
\rho \frac{D \vec{u}}{D t} = \rho \vec{k} - \nabla p,
\]
(2.4)
assuming \( \lambda^* = \eta = 0 \). These restrictions hold for frictionless fluids with the material law
\[
\tau_{ij} = -p \delta_{ij},
\]
(2.5)
where the stress tensor \( \tau_{ij} \) is defined by the pressure, similarly to a resting fluid with the symmetric deformation tensor \( e_{ij} = 0 \), where
\[
e_{ij} = \frac{1}{2} \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right\},
\]
(2.6)

Expression \( \delta_{ij} \) in eq. (2.5) denotes the Kronecker delta.

The assumption of frictionless fluid is a limiting case for fluid flow at large Reynolds numbers
\[
Re = \frac{UL \rho}{\eta} = \frac{UL}{\nu} \rightarrow \infty,
\]
(2.7)
with the velocity $U$ and the typical length $L$, where the forces due to viscosity are very small compared to the forces of inertia.

In this case, the dimensionless Navier-Stokes equation

$$\frac{\partial u^+_i}{\partial t^+} + u^+_j \frac{\partial u^+_i}{\partial x^+_j} + Re^{-1} = -\frac{\partial p^+_i}{\partial x^+_i} + Re^{-1} \frac{\partial^2 u^+_i}{\partial x^+_j \partial x^+_j},$$

(2.8)

which is derived e.g. in [63] and where the superscript $+$ denotes dimensionless parameters, shows that the frictional terms vanish and the Euler equation results.

Considered from a physical point of view, a frictionless fluid flow $\eta \equiv 0$ differs from a frictional flow in the limiting case $\eta \rightarrow \infty$ due to the stiction of the latter at a wall, causing within the boundary layer large velocity gradients and angular velocities. From a mathematical point of view, the distinction lies in the absence of higher order derivatives.

### 2.2. Equations for Free-Surface Flow

Considering the flow of incompressible, non-viscous, non heatconducting water in a channel, the dynamics of the flow can be described by Euler’s equation (2.3) and the continuity equation. With the gravity as body force, acting in $z$-direction, Euler’s equation read

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} - g \delta_{i3}$$

(2.9)

The continuity or mass equation for incompressible fluids is given by

$$\frac{\partial u_i}{\partial x_i} = 0$$

(2.10)

and also derived in appendix A. The resulting system of four equations can be solved for the four unknowns $u, v, w$ and $p$ as functions of space and time, given initial and boundary conditions. The free-surface problem remains difficult to be solved numerically, analytical solutions are impossible to obtain [66]. The main difficulty results from the free surface where boundary conditions have to be satisfied and of which the position is not known, thus the domain for which the equations have to be solved is not unknown.

### 2.3. The Shallow Water Equation

#### 2.3.1. Derivation of the Shallow Water Equations

The shallow water equations approximate the full free-surface problem under the assumption that the velocity field is independent of $z$ and that the vertical velocity component $w$ vanishes: $w = 0$. Thus the $z$-component of the free-surface flow governing equation reads

$$\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g = 0.$$

(2.11)
2.3. The Shallow Water Equation

Following from this expression, the pressure is purely hydrostatic, as eq. (2.11) yields

\[ p = \rho gh = \rho g(s - z) \]  

(2.12)

where the pressure at the free surface \( s \) is the atmospheric pressure and for convenience taken to be identically zero:

\[ p|_{z=s} = p_{atm} = 0. \]  

(2.13)

The last term of eq. (2.12) indicates independency of the pressure from the velocity components \( u \) and \( v \), as its differentiation with respect to \( x \) or \( y \), respectively, gives:

\[ \frac{\partial p}{\partial x} = \rho g \frac{\partial s}{\partial x}, \quad \frac{\partial p}{\partial y} = \rho g \frac{\partial s}{\partial y}. \]  

(2.14)

Both equations show that the derivatives of the pressure are independent of \( z \), therefore the material derivatives of the velocities \( u \) and \( v \) do not depend on this coordinate direction. Hence the equations of motion for the remaining two coordinate directions of the free-surface flow become

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial s}{\partial x} \]  

(2.15a)

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial s}{\partial y}, \]  

(2.15b)

with the constant density dropped out.

The differential conservation law form of the law of conservation of mass follows from integration of the continuity equation (2.10) with respect to the coordinate direction \( z \) between a fixed bottom \( b = b(x, y) \) and the free surface \( s = s(t, x, y) \),

\[ \int_{b}^{s} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \, dz = 0, \]  

(2.16)
Chapter 2. Derivation of the Shallow Water Equation

leading to

\[ w|_{z=s} - w|_{z=b} + \int_b^s \frac{\partial u}{\partial x} \, dz + \int_b^s \frac{\partial v}{\partial y} \, dz = 0. \] (2.17)

The reference system is depicted in fig. 2.1.

As a boundary condition it must be hold

\[ \left( \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} - w \right)|_{z=s} = 0 \] (2.18)

at the free surface and

\[ \left( \frac{\partial b}{\partial t} + u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} - w \right)|_{z=b} = 0, \] (2.19)

at the bottom boundary. Eq. (2.18) and (2.19) correspond to the requirement that no fluxes through the regarded surfaces exist. It follows for the two left-hand side terms in eq.(2.17)

\[ w|_{z=s} = \left( \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} - w \right)|_{z=s} \] (2.20)

and

\[ w|_{z=b} = \left( \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} - w \right)|_{z=b} \] (2.21)

with

\[ \frac{\partial b}{\partial t} = 0. \] (2.22)

The two right-hand terms in eq.(2.17), \( \int_b^s \frac{\partial u}{\partial x} \, dz \) and \( \int_b^s \frac{\partial v}{\partial y} \, dz \), can be written as

\[ \int_b^s \frac{\partial u}{\partial x} \, dz = \frac{\partial}{\partial x} \int_b^s u \, dz - u|_{z=s} \frac{\partial s}{\partial x} - u|_{z=b} \frac{\partial b}{\partial x}, \]

\[ \int_b^s \frac{\partial v}{\partial y} \, dz = \frac{\partial}{\partial y} \int_b^s v \, dz - v|_{z=s} \frac{\partial s}{\partial y} - v|_{z=b} \frac{\partial b}{\partial y}, \] (2.23)

making use of Leibniz’s formula.

From combination of eq. (2.20), (2.21) and (2.23), it follows for eq. (2.17)

\[ \frac{\partial s}{\partial t} + \frac{\partial}{\partial x} \int_b^s u \, dz + \frac{\partial}{\partial y} \int_b^s v \, dz = 0. \] (2.24)

By \( h = s - b \), the independence of the velocity components \( u \) and \( v \) from \( z \) and the constancy in time of the fixed bottom \( b \) this yields

\[ \frac{\partial h}{\partial t} + \frac{\partial u h}{\partial x} + \frac{\partial v h}{\partial y} = 0, \] (2.25)

which is the differential conservation law form of the conservation of mass.

The differential conservation law form of the momentum equations (2.15) is derived as follows:
2.3. The Shallow Water Equation

eq. (2.25) premultiplied by \( u \) and added to (2.15a), premultiplied by \( h \), gives the following form:

\[
\frac{\partial uh}{\partial t} + \frac{\partial u^2 h}{\partial x} + \frac{\partial uh v}{\partial y} = -gh \frac{\partial s}{\partial x};
\]  

(2.26)

analogously (2.25) yields with (2.15b)

\[
\frac{\partial vh}{\partial t} + \frac{\partial uh v}{\partial x} + \frac{\partial v^2 h}{\partial y} = -gh \frac{\partial s}{\partial y}.
\]  

(2.27)

The right-hand side term \(-gh \frac{\partial s}{\partial x}\) can be modified due to

\[s = b + h\quad \text{and} \quad h \frac{\partial h}{\partial x} = \frac{\partial}{\partial x} \left( \frac{1}{2} h^2 \right);\]

(2.28)

this yields

\[-gh \frac{\partial s}{\partial x} = -\frac{1}{2} g \frac{\partial h^2}{\partial x} - gh \frac{\partial b}{\partial x}.
\]  

(2.29)

So the form

\[
\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h + \frac{1}{2} gh^2 \right) + \frac{\partial uh v}{\partial y} = -gh \frac{\partial b}{\partial x}
\]

\[
\frac{\partial vh}{\partial t} + \frac{\partial uh v}{\partial x} + \frac{\partial}{\partial y} \left( v^2 h + \frac{1}{2} gh^2 \right) = -gh \frac{\partial b}{\partial y}
\]

(2.30)

is obtained for the momentum equation.

The one dimensional form with respect to the \( x \)-direction reads

\[
\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h + \frac{1}{2} gh^2 \right) = -gh \frac{\partial b}{\partial x}.
\]  

(2.31)

2.3.2. Restrictions of the Shallow Water Equations for Dam-Break Flows

The derivation of the shallow water equations shows that simplifying assumptions are made. These restrict transferability of the equations to reality.

The shallow water equations base on Euler’s equations which neglects amongst others viscous stresses. These are of interest concerning dissipation of energy and thus loss of momentum, a process that is not taken into account.

Furthermore, negligibly curved stream lines and a hydrostatic pressure distribution are assumed.

Considering dam-break induced flood waves, these assumption do not hold in the wave front and in the initial phase. Thus in this phase and region, respectively, results will not agree with reality.

Due to the limited stream line curvature which is assumed, also turbulence cannot be accounted for.

The shallow water equations model waves with small ratios \( H/L \) where \( H \) denotes the wave height and \( L \) its length. This signifies that waves of small lengths cannot be resolved.

Furthermore, the equations consider the propagation of gravity-induced waves which are driven
Chapter 2. Derivation of the Shallow Water Equation

by the hydrostatic pressure and thus by gravity. For e.g. wind-induced waves additional models are necessary. Depth-averaged velocity-profiles are accounted for. In case of the one dimensional form also horizontal averages result. Considering the dam-breaching process, the following assumptions are made, imposing further restrictions: It is of common practice to assume that the dam breaches instantaneously. This signifies that hydraulic processes are decoupled from the mechanism of failure and the potential energy of the resting fluid is converted suddenly into kinetic energy. Thus the worst case scenario is simulated.

2.3.3. Vector Equation Notation

Another notation of the conservation laws used in the following sections is for the two dimensional shallow water equations the following:

\[
\phi, t + (f(\phi))_x + (g(\phi))_y = \psi(\phi) \tag{2.32}
\]

with

\[
\begin{bmatrix}
\phi \\
\phi_x \\
\phi_y
\end{bmatrix}, \quad \begin{bmatrix}
f(\phi) \\
(f(\phi))_x \\
(f(\phi))_y
\end{bmatrix}, \quad \begin{bmatrix}
g(\phi) \\
(g(\phi))_x \\
(g(\phi))_y
\end{bmatrix}, \quad \begin{bmatrix}
\psi(\phi) \\
\psi(\phi)_x \\
\psi(\phi)_y
\end{bmatrix}
\]

Here \( \phi \) denotes a vector of the considered conserved state variables, \( f(\phi) \) and \( g(\phi) \) are flux function vectors and \( \psi(\phi) \) represents a source term vector. The subscript \( ,x \) denotes a partial differentiation with respect to \( x \) - i.e. \( \cdot, x = \frac{\partial}{\partial x} \); the subscript \( ,y \) is defined analogously.

For the one dimensional system the vector notation is

\[
\phi, t + (f(\phi))_x = \psi(\phi) \tag{2.34}
\]

with the vectors

\[
\phi = \begin{bmatrix}
h \\
hu \\
hv
\end{bmatrix}, \quad f(\phi) = \begin{bmatrix}
hu \\
hu^2 + \frac{1}{2}gh^2 \\
huv
\end{bmatrix}, \quad g(\phi) = \begin{bmatrix}
hv \\
hvu \\
hv^2 + \frac{1}{2}gh^2
\end{bmatrix}, \quad \psi(\phi) = \begin{bmatrix}
0 \\
ghb_x \\
ghb_y
\end{bmatrix}
\]

(2.35)

Under the assumption that \( \phi \) is smooth, a quasi-linear system can be considered. The one-dimensional system thus becomes, with the flux Jacobian matrix \( A = \frac{\partial f}{\partial \phi} \),

\[
\phi, t + A\phi, x = \psi(\phi). \tag{2.36}
\]

In this expression the chain rule was applied, yielding

\[
(f(\phi))_x = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial \phi} \frac{\partial \phi}{\partial x} = A\phi, x. \tag{2.37}
\]
A one-dimensional quasilinear system

\[ \phi_t + A(\phi, x, t)\phi_x = 0 \]  

(2.38)
is hyperbolic at a point \((\phi, x, t)\) if the matrix \(A(\phi, x, t)\) satisfies at this point the hyperbolicity conditions at each physical relevant value of \(\phi\), i.e. if it is diagonalizable with real eigenvalues. These conditions are to be hold for the coefficient matrix \(A\) and any real linear combination of it. The shallow water equations represent a such a system of nonlinear hyperbolic equations. A homogeneous hyperbolic system of partial differential equations results if the source term is assumed to equal zero; from this form analytical solution can be obtained. It is also possible to define additional source terms in the vector \(\psi\); in case of the considered shallow water equation, additionally to the fixed bottom term presented above, terms taking into account effects of friction, viscosity and sediment transport are conceivable or also of wind and Coriolis forces.
3. Discontinuities and Weak Solutions

In contrast to the integral form of the conservation law, the differential one is not valid for shock-containing, hence discontinuous solutions. The consideration of such solutions is enabled by weak forms which will be considered in this section for the one-dimensional case.

The multiplication of a one-dimensional homogeneous differential equation by the test function \( \eta(x,t) \) and its integration over space and time yields

\[
\int_{0}^{\infty} \int_{-\infty}^{+\infty} \eta \phi_{,t} + \eta f(\phi)_{,x} \, dx \, dt = 0
\]  

(3.1)

By integration by part, the weak form of the conservation law is obtained as

\[
\int_{0}^{\infty} \int_{-\infty}^{+\infty} \eta_{,t} \phi + \eta_{,x} f(\phi) \, dx \, dt = -\int_{-\infty}^{+\infty} \eta(x,0) \phi(x,0) \, dx,
\]  

(3.2)

due to the requirement for the test function \( \eta \) to be continuously differentiable with "compact support": Outside of a bounded set, \( \eta(x,t) \) is identical to zero and the support of the function lies in a compact set, so that at infinity the test function vanishes and several boundary terms of the integration by part drop whereas the initial conditions containing term remains.

The form (3.2) requires less smoothness on \( \phi \) on account of the fewer derivatives on it.

If a test function

\[
\eta(x,t) = \begin{cases} 
1 & \text{for } (x,t) \in [x_{1},x_{2}] \times [t_{1},t_{2}] \\
0 & \text{for } (x,t) \notin [x_{1}-\epsilon,x_{2}+\epsilon] \times [t_{1}-\epsilon,t_{2}+\epsilon]
\end{cases}
\]  

(3.3)

is considered, where \( \eta \) is smooth in an intermediate strip \( \epsilon \to 0 \), the form (3.2) approaches the integration of the integral form of the conservation law: Except in the strip the derivatives of the test function equal zero, \( \eta_{,t} = \eta_{,x} = 0 \), hence the integral reduces to the region of the strip.

Assuming for this a vanishing width, the derivatives \( \eta_{,t} \) and \( \eta_{,x} \) will reduce to delta functions which sample the state variables and its flux functions along the boundaries of the rectangle \([x_{1},x_{2}] \times [t_{1},t_{2}]\), corresponding to the integral form. Thus a weak solution satisfies the original integral conservation law [42, p.28].

The mathematical formulation of the physical problem as a hyperbolic equation and the associated weak form imposes the difficulty to find the "right", i.e. a physically admissible, solutions to the weak form: The solutions are not unique by reason of the neglected viscosity. The latter would smoothen the discontinuity of the solution: If viscous effects were taken into consideration, a term \( \eta \phi_{,xx} \) would have to be added to the system of equations, resulting in

\[
\phi_{,t} + f(\phi)_{,x} = \eta \phi_{,xx}.
\]  

(3.4)

This expression represents a parabolic equation and has a unique solution for all time \( t > 0 \) and any set of initial conditions [41, p.211]. The influence of the viscosity term increases if shocks
are formed and hence the derivatives of $\phi$ gain in influence, evoking less sharp solutions in the shock region, see fig. 3.1. If the viscosity approaches zero, a so-called vanishing viscosity solution is achieved; this solution is the scope of working with the inviscid system and constitutes a weak solution to the presented hyperbolic equations. This type of equation is to be preferred compared to the parabolic viscous equations by reason of complicating effects of the latter onto computability, caused by complex eigenvalues of this type of system of equations [42]. Due to the fact that in regions without shock the viscosity term is negligible, the diminishing strip width $\epsilon$ has the same effect to the shape of the solution as the viscosity. Using the hyperbolic system requires thus an additional condition in order to result in the solution's uniqueness and physical admissibility. Following gas dynamics, these are called entropy conditions which ensure in the original problem that the second law of thermodynamics is hold mathematically. With reference to the characteristics solution of the Riemann problem, this signifies that the entropy increasing and thus entropy violating solution depicted in fig. 3.2a across expansion shocks has to be prevented. In this figure characteristics come out of the shock. They build an expansion shock with increasing entropy which is physically not admissible. The realistic solution is a rarefaction fan as shown in fig. 3.2b, it is in contrast to the former stable to perturbations and results if the initial profile is slightly smeared out or if viscosity is taken into account. In adding entropy conditions, it is ensured that physically admissible, "right" solutions to the

\[1\] Concerning the shape of less sharp solutions, it is to keep in mind that in reality shocks are not surfaces of discontinuities: Inside the shock viscous effects and heat conduction occur and the quantities change continuously over a distance which is of order of the mean free path and can be taken as infinitesimally small in technical problems. Thus a rarefaction fan as shown in the figure is the realistic solution.

\[2\] With relation to gas dynamics, expansion shock waves are only possible if the inequality $\left(\frac{\partial^2p}{\partial v^2}\right)_s < 0$ holds; the subscript $s$ denotes a consideration along a constant isentrope. Near the critical point these conditions and thus expansion shock waves can occur.
weak form, are computed. The Lax-entropy condition for a convex scalar conservation law for example reads
\[
f'(\phi_l) > s > f'(\phi_r),
\] (3.5)
where the discontinuity is propagating with speed \(s\) and \(f'(\phi_{l,r})\) is the characteristic speed of the state to the left or right, respectively, of the shock. As in case of convex or concave flux functions a the Rankine-Hugoniot condition satisfying speed \(s\) lies between \(f'(\phi_l)\) and \(f'(\phi_r)\), the entropy condition reduces to
\[
f'(\phi_l) > f'(\phi_r).
\] (3.6)
4. The Riemann Problem of a Dam-Break

In this work, a numerical method based on the methods of characteristics is applied. Topic of section 4.1 is the characteristic structure of a dam-break on a horizontal plane on initially wetted bed for a homogeneous system of the one-dimensional shallow water equations. Furthermore, section 4.2 presents the Riemann solution, the exact solution to the Riemann problem which is imposed by a dam-break.

4.1. The Characteristic Structure of a Dam-Break

From a mathematical point of view, the scenario of a dam-break imposes a Riemann problem: A dam which separates two regions of different water heights bursts at time $t = 0$. The initial waterdepth is given by the piecewise constant function

$$h(x, 0) = \begin{cases} h_l & \text{if } x < 0, \\ h_r & \text{if } x > 0, \end{cases}$$

where $h_l > h_r \geq 0$, and the velocity by

$$u(x, 0) = 0.$$

These initial conditions are pictured in fig. 4.1.

Figure 4.1.: Initial conditions of a dam-break for water height $h$ and momentum $hu$; [41]

The one-dimensional shallow water equations derived in the precedent section describe the proceeding of the waves evoked by a dam-break. For the sake of analytical descriptiveness of this process, it is common practice to consider the homogeneous system of partial differential equations, thus without taking into account viscous or bottom friction for example. System (2.34) then reads

$$\phi_t + (f(\phi))_x = 0$$
with the vectors
\[ \phi = \begin{bmatrix} h \\ hu \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \quad f(\phi) = \begin{bmatrix} hu \\ hu^2 + \frac{1}{2} gh^2 \end{bmatrix}. \] (4.4)

For smooth solutions these equations can be written in the quasilinear form
\[ \phi_t + f'(\phi)\phi_x = 0 \] (4.5)

with the Jacobian
\[ f' = \begin{bmatrix} 0 & 1 \\ -(\phi^2/\phi^1)^2 + g\phi^1 & 2\phi^2/\phi^1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -u^2 + gh & 2u \end{bmatrix}. \] (4.6)

This yields the eigenvalues
\[ \lambda^1 = u - \sqrt{gh}, \quad \lambda^2 = u + \sqrt{gh} \] (4.7)

and the eigenvectors
\[ r^1 = \begin{bmatrix} 1 \\ u - \sqrt{gh} \end{bmatrix}, \quad r^2 = \begin{bmatrix} 1 \\ u + \sqrt{gh} \end{bmatrix}. \] (4.8)

For the nonlinear system presented in eq. (4.3) the eigenvalues and eigenvectors are functions of the conserved variables \( \phi \). They determine the course of the characteristics of the dam-break:

\[ \text{Figure 4.2.: Characteristics of a dam-break in the } x-t\text{-plane; [41]} \]

The characteristics of a dam-break problem with initial data (4.1) and (4.2) are depicted in fig. 4.2 in a \( x-t \)-plane, with the time at the ordinate. The characteristics are shown as thinner lines, the thicker ones depict the waves. The two left-hand sided waves bound the rarefaction fan and build thus the edges of the 1-rarefaction wave, the thick line on the right shows the 2-shock wave. The numbers indicate on which eigenvalue the wave is based. From a physical point of view, the rarefaction wave is a continuous wave, it reduces the free-surface height in a smooth way. For the initial conditions described above, it travels to the left as can be seen in fig. 4.3. The shock
wave is a discontinuous wave, raises the water depth abruptly and travels to the right into the shallow water region. Part a of fig. 4.2 depicts the left-going or 1-characteristics, part b the right-going or 2-characteristics. They show constant slopes where the conserved variables \( h \) and \( hu \) are constant. The 1-characteristics satisfy

\[
\frac{dx}{dt} = \lambda_1 = u - \sqrt{gh}, \tag{4.9}
\]

the first eigenvalue of system (4.5). Inside the rarefaction fan the characteristics spread out, so the wave will spread out and flatten as time evolves. Across the bounding characteristics, all flow variables show a discontinuity in the derivative with respect to the \( x \)-direction. The generalised Riemann invariants are constant across the rarefaction wave and the corresponding eigenvalue increases monotonically in crossing the wave. The shock wave is crossed by the 1-characteristics in steepening their slope which signifies that an acceleration can be observed. The 2-characteristics satisfy the second eigenvalue,

\[
\frac{dx}{dt} = \lambda_2 = u + \sqrt{gh}. \tag{4.10}
\]

The curves cross the rarefaction waves with a smooth change in velocity and impinge on the shock wave what illustrates the compressive and steepening character of this wave.

**Figure 4.3.** Solution of the dam-break Riemann problem for water height \( h \) and momentum \( hu \); [41]
4.2. The Riemann Solution

Solving the Riemann problem described above requires determining how the left and right states \( \phi_l \) and \( \phi_r \) of the conserved variables are connected through the intermediate state \( \phi_m \) across the two waves, i.e. the rarefaction wave and the shock. Therefore the theory of shock waves and rarefaction waves is needed.

From part of the shock waves theory, the so-called Hugoniot locus is used. For the initial conditions given above, this is a curve which represents the set of all possible states \( \phi_m \) in the \( h-u \)-plane that can be connected to the right-hand state by a 2-shock. This is pictured in fig. 4.4 where the state \( \phi_r \) and all in case of a 2-shock possible \( \phi_m \)-states are indicated; the solid line indicates the entropy condition satisfying intermediate states \( \phi_m \). The Riemann solution is derived as follows:

Across any shock the Rankine-Hugoniot condition must be satisfied,

\[
s(\phi_* - \phi_m) = f(\phi_*) - f(\phi_m).
\]  
(4.11)

The subscript \( * \) represents a fixed left or right state \( \phi_l \) or \( \phi_r \) to which the searched state \( \phi_m \) may be connected through a shock.

For the shallow water equations the Rankine-Hugoniot relation reads

\[
s(h_* - h_m) = h_* u_* - h_m u_m
\]

\[
s(h_* u_* - h_m u_m) = h_* u_*^2 - h_m u_m^2 + \frac{1}{2}g(h_*^2 - h_m^2)
\]  
(4.12)

A system of two equations and three unknowns results, thus a one-parameter family of solutions will be found. Usually a parametrization of this family with the water height \( h \) is applied. With the shock speed \( s \) from the first equation in (4.12),

\[
s = \frac{h_* u_* - h_m u_m}{h_* - h_m},
\]  
(4.13)
4.2. The Riemann Solution

the second equation gives

\[ u_m^2 - 2u_su_m + \left[ u_s^2 - \frac{g}{2} \left( \frac{h_s}{h_m} - \frac{h_m}{h_s} \right) (h_s - h) \right] = 0. \]  

(4.14)

The root is

\[ u_m(h_m) = u_s \pm \sqrt{\frac{g}{2} \left( \frac{h_s}{h_m} - \frac{h_m}{h_s} \right) (h_s - h)}, \]

(4.15)

so that each water depth \( h_m \) yields two velocity values \( u_m \) according to the two families of shocks. This equation yields for the right state \( \phi_r \) and connection through a 2-shock an expression which is derived in [41] and reads

\[ u_m = u_r + (h_m - h_r) \sqrt{\frac{g}{2} \left( \frac{1}{h_m} - \frac{1}{h_r} \right)}; \]

(4.16)

for a left state \( \phi_l \) and connection through a 1-shock it is analogously

\[ u_m = u_l - (h_m - h_k) \sqrt{\frac{g}{2} \left( \frac{1}{h_m} - \frac{1}{h_l} \right)}; \]

(4.17)

the Hugoniot locus of this configuration is depicted in fig. 4.5.

![Figure 4.5: Hugoniot locus of points \( \phi_m \) that can be connected to a given state \( \phi_l \) by a 1-shock; [41]](image)

As mentioned above, the second theory necessary to obtain the solution of the Riemann problem is the one of rarefaction waves. From this it is known that the interesting state \( \phi_m \) must lie on an integral curve: If at each point of a curve \( \tilde{\phi}(\xi) \), where \( \xi \) denotes the ratio \( x/t \), the tangent vector \( \tilde{\phi}'(\xi) \) to the curve is an eigenvector of the flux Jacobian \( f'(\tilde{\phi}(\xi)) \) which corresponds to the eigenvalue \( \lambda P(\tilde{\phi}(\xi)) \), the curve is a so-called integral curve. This means that at a given point of the curve, a tangent vector must be a scalar multiple of the eigenvector and thus pointing in the
same direction; a set $r^p(\phi)$ of eigenvectors is determined by the considered system of equations. Hence the expression

$$\tilde{\phi}'(\xi) = \alpha(\xi)r^p(\tilde{\phi}(\xi))$$

(4.18)

is obtained. Figure 4.6 shows the integral curves of an eigenvector field $r^1$; the vectors indicate that the eigenvector $r^1$ evaluated at any point on an integral curve is tangent to it.

![Integral curves of an eigenvector field $r^1$](image)

**Figure 4.6.:** Integral curves of an eigenvector field $r^1$; [41]

For the shallow water equations, consideration of the $r^1$-field with $\alpha(\xi) = 1$ yields the system

$$\tilde{\phi}'(\xi) = r^1(\tilde{\phi}(\xi)) = \begin{bmatrix} 1 \\ \tilde{\phi}^2/\tilde{\phi}^1 - \sqrt{gh} \end{bmatrix},$$

(4.19)

so that the ordinary differential equations

$$\begin{align*}
(\tilde{\phi}^1)' &= 1 \\
(\tilde{\phi}^2)' &= \tilde{\phi}^2/\tilde{\phi}^1 - \sqrt{gh} \\
\end{align*}$$

(4.20)

are obtained which are satisfied for $\tilde{\phi}^1(\xi) = \xi$. So the integral curve will be parametrized by the depth which is the first component of $\phi$. The second ODE becomes hence

$$\begin{align*}
(\tilde{\phi}^2)' &= \tilde{\phi}^2/\xi - \sqrt{gh}. \\
\end{align*}$$

(4.21)

This equation can be solved with a fixed point $(h_*, u_*)$ and with $\tilde{\phi}^2(\xi) = h_*u_*$. The solution reads

$$\tilde{\phi}^2(\xi) = \xi u_* + 2(\sqrt{gh} - \sqrt{gh}).$$

(4.22)

In terms of the velocity and inserting the depth for $\xi$, the integral curve of $r^1$ which passes through the fixed state $(h_*, u_*)$ has the form

$$u = u_* + 2(\sqrt{gh} - \sqrt{gh}).$$

(4.23)
for integral curves of $r^2$, shown in fig. 4.7, it is

$$ u = u_* - 2(\sqrt{gh_*} - \sqrt{gh}). \quad (4.24) $$

By superimposing the Hugoniot locus and the integral curve the intermediate state $\phi_m$ is deter-

![Figure 4.7: Integral curves of an eigenvector field $r^2$; [41]](image)

mined. The equations (4.16) and (4.23) give thus together the state $h_m$ obtained from the left state through a 1-rarefaction and from the right state through a 2-shock; as the 1-wave being a shock and the 2-wave a rarefaction yield also a possible solution to different initial conditions, eq. (4.17) and (4.24) yield combined another possible intermediate state. Thus the following system of two functions $g_l$ and $g_r$ can be defined:

$$ g_l = \begin{cases} 
    u_l + 2(\sqrt{gh_l} - \sqrt{gh_m}) & \text{if } h < h_l, \\
    u_l - (h_m - h_l)\sqrt{\frac{g}{2} \left( \frac{1}{h_m} + \frac{1}{h_l} \right)} & \text{if } h > h_l,
\end{cases} \quad (4.25) $$

and

$$ g_r = \begin{cases} 
    u_r - 2(\sqrt{gh_r} - \sqrt{gh_m}) & \text{if } h < h_r, \\
    u_r + (h_m - h_r)\sqrt{\frac{g}{2} \left( \frac{1}{h_m} + \frac{1}{h_r} \right)} & \text{if } h > h_r.
\end{cases} \quad (4.26) $$

They ensure physically correct solutions which do not violate the entropy conditions. Function (4.25) can be redefined for a 1-wave connection between the fixed state $\phi_l$ and the intermediate state as

$$ f_l(h, h_l) = \begin{cases} 
    2(\sqrt{gh} + \sqrt{gh_l}) & \text{if } h < h_l, \\
    (h - h_l)\sqrt{\frac{g}{2} \left( \frac{1}{h} + \frac{1}{h_l} \right)} & \text{if } h > h_l.
\end{cases} \quad (4.27) $$
with
\[ u(h) = u_l - f_l(h, h_l). \] (4.28)

Analogously function (4.26) yields for the case of a 2-wave
\[
f_r(h, h_r) = \begin{cases} 
2(\sqrt{gh} - \sqrt{gh_r}) & \text{if } h < h_r, \\
(h - h_r)\sqrt{\frac{g}{2} \left( \frac{1}{h} + \frac{1}{h_r} \right)} & \text{if } h > h_r.
\end{cases}
\] (4.29)

with
\[ u(h) = u_r + f_r(h, h_r). \] (4.30)

Eq. (4.28) and (4.30) build curves in phase space; at their intersection the unknown intermediate state must occur. By subtraction of both equations the function \( f \)
\[
f(h) = f_l(h, h_l) + f_r(h, h_r) + \underbrace{u_r - u_l}_{\Delta u} \quad h_m > 0.
\] (4.31)

is obtained. The root of this expression yields the intermediate state \( h_m \):
\[
f(h_m) = f_l(h_m, h_l) + f_r(h_m, h_r) + \Delta u = 0;
\] (4.32)

the velocity \( u_m \) results from eq. (4.28) or (4.30) according to the initial conditions.

The complete Riemann solution of the dam-break problem still requires the structure of the rarefaction wave connecting the fixed state to the middle state.

Fig. 4.8 shows with solid lines a Hugoniot locus of a 2-shock and an integral curve of an 1-rarefaction, intersecting at the middle state \( \phi_m \).

\[ \text{Figure 4.8.: Integral curves of an eigenvector field } r^2; \] [41]

\[ ^1 \text{The dashed lines denote Hugoniot locus and integral curve for an entropy-violating 1-shock and an unphysical 2-rarefaction.} \]
5. Numerical Methods

A wave-propagation algorithm applicable to Riemann problems is the Godunov method presented in section 5.1. Section 5.2 presents computationally less expansive approximate solvers which can be regarded as being based on Godunov’s scheme; the Roe linearization, sonic entropy fixes and, resulting from these, the HLL and HLLE solver are considered. The more accurate and less diffusive high-resolution methods are discussed in section 5.3. Content of the closing section 5.4 are the calculation of the Riemann problem in case of initially dry beds and the preservation of positive values for the water height; these are difficulties occurring due to the numerical treatment of the shallow water equations.

5.1. The Godunov Method

The Godunov method is a forwards in time Riemann problems solving method using the characteristic information. The solution of the Riemann problem is utilised locally, the solution can be exact or approximate. It is a generalization of the first-order upwind method to conservation law systems; the basic scheme is first-order accurate in space and time, higher-order extensions are possible. In this section the wave propagation form of the Godunov Method is presented.

The average of the state variable $\phi(x, t_n)$ at time $t_n$ in the cell $i$ is considered, it is denoted by the quantity $\Phi_i^n$

$$\Phi_i^n \approx \frac{1}{\Delta x} \int_{x_{i-1/2}}^{x_{i+1/2}} \phi(x, t_n) \, dx. \quad (5.1)$$

From this cell average a function $\bar{\phi}(x, t_n)$ for the shape of the variable in the cell is defined; in the simplest case it is piecewise constant, thus

$$\bar{\phi}(x, t_n) = \Phi_i^n. \quad (5.2)$$

The basic idea is that the cell average for a new time step is affected by a set of waves. Each wave $W$ consists of a jump in $\phi$ which is computable as a scalar multiple of an eigenvector $r$, hence across the $p$-th wave the jump can be expressed as

$$W^p = \alpha^p r^p. \quad (5.3)$$

The jump propagates at the velocity $\lambda^p$, the $p$-th eigenvalue of the system; after an increment $\Delta t$ of time is has moved a distance $\lambda^p \Delta t$ and enters a cell of width $\Delta x$ up to the ratio $\lambda^p \Delta t / \Delta x$. At this fraction of the cell the average value is modified by the propagating wave. Therefore an updated average cell value, influenced by several waves defined at the neighbouring cell interfaces
Chapter 5. Numerical Methods

$i - 1/2$ and $i + 1/2$ and travelling with positive velocities $\lambda^+$ as right going wave from $i - 1/2$ or with negative velocities $\lambda^-$ as left going wave from $i + 1/2$, is obtained by

$$
\Phi_{i}^{n+1} = \Phi_{i}^{n} - \frac{\Delta t}{\Delta x} \left[ \sum_{p} (\lambda^{p})^+ W_{i-1/2}^{p} + \sum_{p} (\lambda^{p})^- W_{i+1/2}^{p} \right].
$$

(5.4)

This expression represents a generalization of a first order upwind method.

Introducing for the change in $\Phi_{i}^{n}$ through left- or right-going waves, respectively, the symbols

$$
A^+ \Delta \Phi_{i+1/2} = \sum_{p} (\lambda^{p})^+ W_{i-1/2}^{p} = \sum_{p} (\lambda^{p})^+ \alpha_{i-1/2}^{p} r^{p}
$$

(5.5)

$$
A^- \Delta \Phi_{i-1/2} = \sum_{p} (\lambda^{p})^- W_{i+1/2}^{p} = \sum_{p} (\lambda^{p})^- \alpha_{i+1/2}^{p} r^{p},
$$

eq. (5.4) takes the form

$$
\Phi_{i}^{n+1} = \Phi_{i}^{n} - \frac{\Delta t}{\Delta x} \left[ A^+ \Delta \Phi_{i+1/2} + A^- \Delta \Phi_{i-1/2} \right].
$$

(5.6)

The symbol $A^+ \Delta \Phi_{i+1/2}$ can be interpreted as the net effect of all right-going waves from the cell interface $x_{i-1/2}$ measuring single entity; analogously $A^- \Delta \Phi_{i-1/2}$ measures the effect of all left-going waves from the cell interface $x_{i+1/2}$.

5.2. Approximate Riemann Solvers

Approximate Riemann solvers do not determine the entire structure of the Riemann problem but an approximating state $\hat{\Phi}_{i-1/2}(x,t)$ based on given data $\Phi_{i-1/2}$ and $\Phi_i$. This is considered to be sufficient with respect to the fact that the application of Godunov’s method does not require the entire solution of the Riemann problem, e.g. the exact solution is averaged over each grid cell.

The approximate Riemann solution can be obtained through replacing the nonlinear problem by a linearized one which is defined locally at each cell interface,

$$
\hat{\phi}_{t} + \hat{A}_{i-1/2} \hat{\phi}_{x} = 0.
$$

(5.7)

The matrix $\hat{A}_{i-1/2}$ represents as a local linearization of the flux Jacobian $f'(\phi)$ an approximation to the latter. It is valid in a neighbourhood of the data $\Phi_{i-1/2}$ and $\Phi_i$ and replaces the Jacobian; the resulting linear Riemann problem is solved with respect to this approximation. As in the case of a linear Riemann problem, the solution will consist of a set of waves propagating at speeds which are calculated through the eigenvalues of the constant matrix of the flux derivatives. A distinction to the circumstances of linear Riemann problem lies in the number of waves which can now differ from the number of equations. The wave speeds are not necessarily identical to
5.2. Approximate Riemann Solvers

the eigenvalues as well. A conservative, i.e. conservation law obeying, approximate solution will satisfy

\[ f(\phi_r) - f(\phi_l) = \sum_p s^p W^p, \] (5.8)

which means that flux evaluated at the far sides of the changing region equals the by the jump discontinuity caused time rate of change of the solution [32].

5.2.1. The Roe Linearization

The locally at each cell interface defined approximation \( \hat{A}_{i-1/2} \) of the Jacobian matrix has to satisfy the following points:

1. it must be diagonalizable with real eigenvalues, yielding a hyperbolic system of equations
2. it must converge to the original flux Jacobian,

\[ \hat{A}_{i-1/2} \to f'(\bar{\phi}) \quad \text{as} \quad \Phi_{i-1}, \Phi_i \to \bar{\phi}, \] (5.9)

so that consistency with the original conservation law is hold

3. The condition

\[ \hat{A}_{i-1/2}(\Phi_i - \Phi_{i-1}) = f(\Phi_i) - f(\Phi_{i-1}) \] (5.10)

must be hold. This expression results from satisfaction of two other conditions:

- the Rankine-Hugoniot relation

\[ f(\Phi_i) - f(\Phi_{i-1}) = s(\Phi_i - \Phi_{i-1}) \] (5.11)

which applies to the connection of two states \( \Phi_{i-1} \) and \( \Phi_i \) through a single wave must be hold. This means that if in the true Riemann solution the two states are connected by a single wave \( W^p = \Phi_i - \Phi_{i-1} \), then this wave should be an eigenvector of the approximated matrix \( \hat{A}_{i-1/2} \).

- a possible solution to the linearized Riemann problem is looked for, so

\[ \hat{A}_{i-1/2}(\Phi_i - \Phi_{i-1}) = s(\Phi_i - \Phi_{i-1}) \] (5.12)

has to be satisfied

By insertion eq. (5.10) follows.

A matrix satisfying the last condition can be obtained by integration of the Jacobian matrix over some path in state space between the states \( \Phi_{i-1} \) and \( \Phi_i \). A formulation for this path has to take into account the difficulty that an evaluation of the integration has to be possible and that the resulting matrix \( \hat{A}_{i-1/2} \) must still hold the first condition.

Roe introduced a change of variables leading to integrals that are easy to evaluate [41]. For
this a parameter vector $z(\phi)$ is utilized so that the function $f$ is a function of $z$; furthermore the parameter vector is assumed to be an invertible mapping so that $\phi(z)$ is known. With $Z_j$ denoting $z(\Phi_j)$, the path along which will be integrated is given by

$$z(\xi) = Z_{i-1} + (Z_i - Z_{i-1})\xi$$

(5.13)

for $0 \leq \xi \leq 1$; the deviation $z'(\xi) = Z_i - Z_{i-1}$ is independent of $\xi$. Now $f(\Phi_i) - f(\Phi_{i-1})$ can be written as line the integral

$$f(\Phi_i) - f(\Phi_{i-1}) = \int_0^1 \frac{df(z(\xi))}{dz} d\xi$$

(5.14)

$$= \int_0^1 \frac{df(z(\xi))}{dz} z'(\xi) d\xi$$

(5.15)

$$= \left[ \int_0^1 \frac{df(z(\xi))}{dz} d\xi \right] Z_i - Z_{i-1}.$$  

(5.16)

Applying the same path integral to $\Phi_i - \Phi_{i-1}$ relates this expression to $Z_i - Z_{i-1}$:

$$\Phi_i - \Phi_{i-1} = \int_0^1 \frac{d\phi(z(\xi))}{dz} d\xi$$

(5.17)

$$= \left[ \int_0^1 \frac{df(\phi(\xi))}{dz} d\xi \right] Z_i - Z_{i-1}.$$  

(5.18)

With the symbols $\hat{B}_{i-1/2}$ and $\hat{C}_{i-1/2}$ introduced for the integrals in eq. (5.16) and (5.16) it follows for the third condition, eq. (5.10), the expression

$$\hat{A}_{i-1/2} = \hat{C}_{i-1/2}\hat{B}_{i-1/2}^{-1}.$$  

(5.20)

The matrix $\hat{A}_{i-1/2}$ can be evaluated if a parameter vector $z(\phi)$ is found that is assessable for both integrals $\hat{B}_{i-1/2}$ and $\hat{C}_{i-1/2}$. This is achieved in defining the parameter vector in such a way that both derivatives $\partial\phi/\partial z$ and $\partial f/\partial z$ include polynomials in the components of $z$.

According to [41] who cites Harten and Lax\(^1\) the obtained matrix $\hat{A}_{i-1/2}$ satisfies the first condition, the diagonalizability, if the system has a convex entropy function $\eta(\phi)$.

5.2.2. Sonic Entropy fixes

In case of transonic rarefactions Godunov-type methods will not necessarily converge to the correct, discontinuous, entropy satisfying weak solution. In this type of rarefaction the fluid is

5.2. Approximate Riemann Solvers

accelerated from sub- to supersonic velocities in gas dynamics, or in the considered water streams from sub- to supercritical velocities with respect to the Froude number. The characteristics will be both left- and rightgoing and cross the stagnation or sonic point where the fluid speed equals the sonic speed, see fig. 5.1; hence the value of the state variable propagates with velocity zero.

![Figure 5.1.](image)

**Figure 5.1.** Characteristics for a transonic rarefaction wave; [41]

In order to handle transonic rarefactions properly, entropy fixes are used, ensuring that no entropy violating solutions, e.g. an expansion shock\(^2\), are calculated and that Godunov’s method converges to the correct solution. Due to insufficient numerical viscosity of this numerical method if wave speeds are very close to zero, there may remain a small expansion shock of magnitude \(O(\Delta x)\), a so-called entropy-glitch, which will vanish as the grid is refined. In contrast to the physical viscosity the numerical depends on the local Courant number since it results from the averaging process. Fig. 5.2 shows results for the computation of a transonic rarefaction wave with and without entropy fix and the entropy glitch.

![Figure 5.2.](image)

**Figure 5.2.** Analytical (solid line) and numerical (circles) solution for a transonic rarefaction wave; [41]

\(^2\)Approximate solvers represent solutions of these waves spreading out in both directions through a single discontinuity, an expansion shock.
5.2.2.1. The Harten-Hyman Entropy Fix

The entropy fix presented by Harten and Hyman constitutes one possibility to ensure physically admissible solutions to transonic Riemann problems. It considers a transonic rarefaction in the $k$-wave where $\lambda^l < 0 < \lambda^r$, with $\lambda^l$, as $k$th eigenvalue of the flux derivative matrix of the states directly to the left or right, respectively, of the considered wave. The states of the conserved variables read

$$\phi_i^k = \Phi_{i-1} + \sum_{p=1}^{k-} \mathcal{W}_p, \quad \phi_i^k = \phi_i^k + \mathcal{W}_i^k,$$

(5.21)

where the subscripts $i-1/2$ were neglected for the sake of clarity. The at the speed $\hat{\lambda}^k$ propagating wave $\mathcal{W}_i^k$ can be replaced by a pair of waves $\mathcal{W}_l^k$ and $\mathcal{W}_r^k$, this corresponds to a split of the total jump into two smaller ones:

$$\hat{\lambda}^k \mathcal{W}_i^k = \lambda^l \mathcal{W}_l^k + \lambda^r \mathcal{W}_r^k.$$

(5.22)

The replacing pair of waves is given by

$$\mathcal{W}_l^p = \beta \mathcal{W}_l^k \quad \mathcal{W}_r^p = (1 - \beta) \mathcal{W}_r^k,$$

(5.23)

where the averaged eigenvalue is calculated by

$$\beta = \frac{\lambda^l - \hat{\lambda}^k}{\lambda^r - \lambda^l}.$$

(5.24)

The changes $A^\pm \Delta \Phi_{i-1/2}$ in the state variable can thus computed via eq. (5.5) with, for the $k$th field,

$$(\hat{\lambda}^k)^- = \beta \lambda^l \quad (\hat{\lambda}^k)^+ = (1 - \beta) \lambda^r.$$

(5.25)

5.2.2.2. The LLF Entropy Fix

The LLF entropy fix combines an approximate Riemann Solver with an extension of the local Lax-Friedrichs (LLF) method and introduces in this way more numerical viscosity to the solution. The expression $A^\pm \Delta \Phi_{i-1/2}$ defined in eq. (5.5) is replaced by

$$A^- \Delta \Phi_{i-1/2} = \frac{1}{2} \sum_p \hat{\lambda}^p_{(i-1/2)} \mathcal{W}_{i-1/2}^p,$$

$$A^+ \Delta \Phi_{i-1/2} = \frac{1}{2} \sum_p \hat{\lambda}^p_{(i-1/2)} + a^p_{(i-1/2)} \mathcal{W}_{i-1/2}^p,$$

(5.26)

where

$$a^p_{(i-1/2)} = \max \left[ \left| \lambda^p_{(i-1)} \right|, \left| \lambda^p_{(i)} \right| \right].$$

(5.27)

This formulation results in adding numerical viscosity to all fields, independent of the presence of a transonic rarefaction. It is to note that for smooth solutions the Roe-eigenvalue approaches the eigenvalues of the Jacobian, $\hat{\lambda}^p_{(i-1)} \approx \lambda^p_{(i)} \approx \lambda^p$, so that eq. (5.26) essentially reduces to the standard definition of $A^\pm \Delta \Phi_{i-1/2}$ [41, p.327].
5.3. High-Resolution Methods

5.2.3. The HLL and the HLLE Solver

The by Harten, Lax and van Leer (HLL) proposed approximate Riemann solver estimates the solution to a middle state between two discontinuities $\hat{\Phi}$ based on only two waves propagating at the speeds $s_{i-1/2}^1$ and $s_{i-1/2}^2$. The waves

$$\mathcal{W}_{i-1/2}^1 = \hat{\Phi}_{i-1/2} - \Phi_{i-1} \quad \mathcal{W}_{i-1/2}^2 = \Phi_i - \hat{\Phi}_{i-1/2}$$

(5.28)

yield for the requirement of a conservative solution

$$s_{i-1/2}^1(\hat{\Phi}_{i-1/2} - \Phi_{i-1}) + s_{i-1/2}^2(\Phi_i - \hat{\Phi}_{i-1/2}) = f(\Phi_i) - f(\Phi_{i-1}).$$

(5.29)

The state $\hat{\Phi}_{i-1/2}$ can hence be computed by

$$\hat{\Phi}_{i-1/2} = \frac{f(\Phi_i) - f(\Phi_{i-1}) - s_{i-1/2}^2 \Phi_i + s_{i-1/2}^1 \Phi_{i-1}}{s_{i-1/2}^1 - s_{i-1/2}^2}.$$  

(5.30)

The HLL solver is identical the Roe solver if the wave speeds correspond to the Roe eigenvalues [32, p.23].

An extension to the HLL solver was proposed by Einfeldt who determines the wave speeds by

$$s_{i-1/2}^1 = \min_p \left[ \min \left[ \lambda_{i-1}^p, \hat{\lambda}_{i-1/2}^p \right] \right],$$

$$s_{i-1/2}^2 = \max_p \left[ \max \left[ \lambda_{i+1}^p, \hat{\lambda}_{i-1/2}^p \right] \right],$$

(5.31)

resulting in the HLLE solver. A disadvantageous property of this type of solver is that only two waves model the Riemann solution and that these are determined by approximate speeds of the slowest and fastest waves of the system; if systems of more waves are considered, waves with intermediate speeds may be insufficiently resolved.

5.3. High-Resolution Methods

The in the precedent sections presented Godunov method and its related methods are only first-order accurate and give raise to numerical diffusion. They can be extended to so-called high-resolution methods which combine first-order Godunov methods in regions of shocks or steep gradients with second-order methods where the solution behaves smoothly. The exclusive utilisation of e.g. second-order accurate methods such as the one by Lax-Wendroff is not possible since they fail near discontinuities where oscillations are generated and because their dispersive nature results in inaccurate solutions in case of steep gradients. High-resolution methods instead apply second-order corrections to first-order methods and limiter functions which measure the local smoothness of the solution.
An extension of Godunov’s wave propagation method according to eq. (5.6) can be written as
\[
\Phi_{i}^{n+1} = \Phi_{i}^{n} - \frac{\Delta t}{\Delta x} \left[ A^+ \Delta \Phi_{i+1/2} + A^- \Delta \Phi_{i-1/2} \right] + \frac{\Delta t}{\Delta x} \left[ \tilde{F}_{i+1/2} + \tilde{F}_{i-1/2} \right],
\]  
with \( A^\pm \Delta \Phi_{i-1/2} \) as the fluctuations of Godunov’s method or one of its variants and where \( \tilde{F}_{i\pm1/2} \) denote the limited second-order correction fluxes. These are given by
\[
\tilde{W}_{i-1/2}^p = \frac{1}{2} \sum_{p=1}^{M_w} |s_{i-1/2}^p| \left[ 1 - \frac{\Delta t}{\Delta x} |s_{i-1/2}^p| \right] \tilde{W}_{i-1/2}^p,
\]  
\( \tilde{W}_{i-1/2}^p \) denoting a limited wave,
\[
\tilde{W}_{i-1/2}^p = \gamma \tilde{W}_{i-1/2}^p.
\]  
In this expression, \( \gamma \) is a limiter function. These are defined through an additional parameter \( \kappa_{i-1/2}^p \) which measures the local smoothness in a \( p \)th wave field near \( x_{i-1/2} \) through comparison of the jump in the solution in adjacent grid cells:
\[
\kappa_{i-1/2}^p = \frac{\tilde{W}_{i-1/2}^p \tilde{W}_{i-1/2}^p}{|\tilde{W}_{i-1/2}^p|}.
\]  
The index \( I \) represents the upwind side of the considered cell interface,
\[
I = \begin{cases} 
  i - 1 & \text{if } \lambda_p > 0, \\
  i + 1 & \text{if } \lambda_p < 0,
\end{cases}
\]  
thus for smooth data the variable \( \kappa_{i-1/2}^p \) will approach unity, at discontinuities it will differ from this considerably, e.g. for \( \tilde{W}_{i-1/2}^p \gg \tilde{W}_{i-1/2}^p \kappa_{i-1/2}^p \) is small. With this parameter the flux limiter function is defined as \( \gamma = \gamma(\kappa) \) and hence depending on smoothness. For unsmooth solutions in the \( p \)th wave field, the limiter function should approach zero. Some high-resolution limiters are defined as
\[
\begin{align*}
\text{minmod:} & \quad \gamma(\kappa) = \text{minmod} [1, \kappa] \\
\text{superbee:} & \quad \gamma(\kappa) = \max [0, \min (1, 2\kappa), \min (2, \kappa)] \\
\text{MC:} & \quad \gamma(\kappa) = \max [0, \min \left( \frac{1+\kappa}{1+|\kappa|}, 2, 2\kappa \right)] \\
\text{van Leer:} & \quad \gamma(\kappa) = \frac{\kappa + |\kappa|}{1 + |\kappa|}.
\end{align*}
\]  
Most of these high-resolution limiters are variations between several linear methods, which are for example
\[
\begin{align*}
\text{upwind:} & \quad \gamma(\kappa) = 0 \\
\text{Lax-Wendroff:} & \quad \gamma(\kappa) = 1 \\
\text{Beam-Warming:} & \quad \gamma(\kappa) = \theta \\
\text{Fromm:} & \quad \gamma(\kappa) = \frac{1}{2} (1 + \kappa).
\end{align*}
\]
The MC limiter switches between the first-order Godunov method ($\gamma = 0$), the Lax-Wendroff method ($\gamma = 1$) and Fromm’s method ($\gamma = 2$).

A major requirement on limiter functions is their total variation diminishing (TVD) behaviour. The total variation of a function measures the oscillations in its solution, e.g.

$$TV(\Phi) = \sum_{i=-\infty}^{\infty} |\Phi_i - \Phi_{i-1}|;$$

(5.39)

LeVeque [41] presents some further definitions. If a method introduces oscillations to a numerical solution the total variation of $\Phi^n$ will increase with time; an oscillations-avoiding method should thus not increase the parameter TV,

$$TV(\Phi^{n+1}) \leq TV(\Phi^n);$$

(5.40)

a property that is termed by total variation diminishment and that is fulfilled by the presented high-resolution flux limiter functions, see [41] and fig. 5.3 where the shapes of both the linear and the high-resolution limiters in a $\kappa$-$\gamma$-plane are shown. The local smoothness $\kappa$ is denoted as $\theta$ and the value $\gamma$ of the limiter function as $\phi$. The shaded regions indicate where the function values must lie in order to show properties of a TVD method. For discussions and more detailed presentations of this requirement and the different limiter functions it is refered to [41], for example. Differences between the limiters occur amongst others in the dependence of the accuracy on the grid spacing.
Figure 5.3.: Limiter functions in a $\kappa$-$\gamma$-plane, here denoted as $\theta$-$\phi$-plane, with indicated TVD-region; [41]
5.4. Numerical Difficulties

The transformation of the mathematical problem into a numerical one gives, due to the applied
discretisation, raise to two problems: The preservation of positive water heights and the computa-
tion on initially dry beds. Both cases, water heights lower Zero and identically Zero, change
the hyperbolic system of equations into an elliptic or parabolic one, respectively.

5.4.1. Depth Positivity

Basing on the functions (4.25) and (4.26) presented in the precedent section, a so-called depth
positivity condition can be derived whose relevance is founded by the physical necessity of non-
negative water depths values or, in gas dynamics, of positive densities. In the latter application
area this topic is known as the vacuum state problem.

According to [65], the depth function (4.31) is a monotonically increasing function of
\( h \) and furthermore the three following solutions are possible, depending on
\( h_{\text{min}} = \min[h_l, h_r] \) and \( h_{\text{max}} = \max[h_l, h_r] \):

Case 1 - two rarefactions: \( f(h_{\text{min}}) \geq 0 \)
Case 2 - one rarefaction, one shock: \( f(h_{\text{max}}) \geq 0 > f(h_{\text{min}}) \) \hspace{1cm} (5.41)
Case 3 - two shocks: \( f(h_{\text{min}}) < 0. \)

The shape of the function depends on the fixed states of the water depth and the difference in
the velocities, \( \Delta u \) shifts the curves up or down along the ordinate and determines thus the height
of the intermediate state, i.e. the root of the function. Hence it is possible that for a value of \( \Delta u \)
a negative-signed root \( h \) results which is from a mathematical point of view not possible since
the solution set (4.31) is restricted by \( h \geq 0 \); from a physical point of view negative water depths
are clearly excluded. The critical case of \( h_m = 0 \) corresponds to

\[ f(h_m = 0) = 2(\sqrt{gh_l} + \sqrt{gh_r}) - \Delta u = 0, \] \hspace{1cm} (5.42)

which can be obtained from eq. (4.32) with (4.27) and (4.29). consequently the following condition
is obtained

\[ (\Delta u)_{\text{crit}} \equiv 2(\sqrt{gh_l} + \sqrt{gh_r}) > 0, \] \hspace{1cm} (5.43)

which is called the depth positivity condition.

5.4.2. Dry Bed

The adjacency of a dry region with a water depth of zero to a wet region changes the structure
of the solution of the Riemann Problem. In contrast to the situation of two neighbouring regions
with different, but non-zero water depths, where a shock and a rarefaction wave will occur, only
one wave family can develop in case of a dry bed region which is connected to a wet region. This
results from the impossibility that adjacent to a situation of \( h_0 = 0 \) a shock wave forms which
can be proved as follows\(^3\): If a left state with \( h_l > 0 \) and an arbitrary velocity \( u_l \) is considered, neighbouring a right dry state \( h_0 = 0 \) and some velocity \( u_0 \), under the assumption of a shock wave which connects at a wave speed \( S \) both states, the Rankine-Hugoniot conditions yields

\[
\begin{align*}
    h_l u_l &= h_0 u_0 + s(h_l - h_0) \\
    h_l u_l^2 + \frac{1}{2} g h_l^2 &= h_0 u_0^2 + \frac{1}{2} g h_0^2 + S(h_l u_l - h_0 u_0).
\end{align*}
\]

Following from the dry state water depth \( h_0 = 0 \) and the first equation, the propagation speed of the assumed shock wave is identical to the fluid velocity of the left state, \( S = u_l \). If this relation is inserted in the second equation, a left state water depth of \( h_l = 0 \) results. This is contradictory to the assumption of a wet left state, \( h > 0 \), proving the statement of the non-forfeitability of a shock wave and a dry bed state. Instead, the region of zero water depth is connected to the wet front by a contact discontinuity, i.e. no characteristics reach into the neighbouring region.

From the Riemann invariant across a 1-wave, which is to apply to the right dry bed situation presented above,

\[ u + 2a = \text{constant}, \]

it follows

\[ u_c + 2a_c = u_l + 2a_l, \]

where the subscript \( c \) denotes states along the contact discontinuity. These are \( h_c = 0 \) and \( a_c = 0 \), thus the speed of the contact discontinuity adjacent to the dry bed or of the characteristic coinciding with the front is

\[ S_l^* = u_c = u_l + 2a_l. \]

For the case of a right dry bed a 2-wave connects both states and accordingly to the Riemann invariant

\[ u - 2a = \text{constant} \]

the wet/dry front propagates at the speed

\[ S_r^* = u_c = u_r - 2a_r. \]

\(^3\)Compare [65]
6. Aspects of Sediment Transport

This thesis will not give a detailed presentation of sediment transport models and their derivation or of sediment transport influencing factors like e.g. ripples and dunes: In the specialized literature a multitude of works concerning these topics can be found and for the aim of this thesis it seems to be more interesting and to correspond better to the actual focus of the art to examine phenomena of sediment transport related to steep slopes and dam-breaks like the so-called debris flows. A description of the physical causal relationships of and observable events during these debris flows is given in section 6.2. The to that one precedent section 6.1 will touch different kinds of sediment transport models for mild and steep slopes. Section section 6.3 presents the bed-evaluation equation. Section B of the appendix deals with the initiation of motion as a precondition of sediment transport.

6.1. Sediment Transport Models

This section overviews some of the most frequently utilized models for bed-load transport in form of the formulas by Meyer-Peter and Mueller and by Van Rijn for mild slopes and of the formulas by Smart and Jaeggi and by Rickenmann for steeper slopes.

6.1.1. Mild Slopes

The transport processes in case of rivers of comparatively mild slopes up to 3% were intensively investigated during the last about 80 years and are more or less well-described by several empirical transport models. Those distinguish between bed-load transport, i.e. transport of sediment particles in a thin layer\(^1\) over the ground in sliding, rolling or saltating movements on the one hand and, on the other hand, the suspended-load case where the transport of in the water column suspended particles is considered. Also a combination of both transport processes is possible, e.g. through so-called total-load transport models.

Amongst the formulas of common practice range those of Meyer-Peter and Mueller and Van Rijn which will be presented as an example for deterministic bed load transport models. For a more detailed presentation of these and further models which also make use of different, e.g. stochastic approaches it is again referenced to [67].

The formula of Meyer-Peter and Mueller was developed through experimental work with slopes up to 2 % and bed material of diameters up to 29\( mm\); uniform material as well as particle mixtures were used. The formula of 1949 is given by

\[
q_s = 8 \sqrt{(\rho_s - \rho) / \rho g d_m^3 \mu (\theta - \theta_{cr})^3},
\]  

(6.1)

\(^1\)e.g. two particle diameters of thickness according to Einstein
where \( q_s \) denotes the volumetric bed-load transport rate in \([m^2/s]\), \( \rho_s \) the density of the sediment whereas \( \rho \) describes the density of the water; \( d_{m} \) indicates the mean particle diameter\(^2\) and the parameter \( \mu \) a bed-form or efficiency factor,

\[
\mu = \sqrt{\left( \frac{C}{C'} \right)^3}. \tag{6.2}
\]

This expression is the ratio of the overall Chézy-coefficient \( C \) and the grain-related Chézy-coefficient \( C' \) with

\[
\begin{align*}
C &= 18 \log \left( \frac{12h}{k_s} \right) \quad \text{and} \quad C' &= 18 \log \left( \frac{12h}{d_{90}} \right), \tag{6.3}
\end{align*}
\]

in which \( h \) indicates the water depth, \( k_s \) the effective bed roughness and \( d_{90} \) the particle diameter at 90% of mass passing; this parameter is often approximated as \( d_{90} = 3d_{m} \).

The transport rate computed with formula eq. (6.1) depends directly on the difference between the critical shear stress and the effective shear stress. A different approach was chosen by Van Rijn in his transport model 1984a, where he followed Bagnold relating the bed-load transport rate to the saltation height \( \delta_b \), the bed-load concentration \( c_b \) and the particle velocity \( u_b \) as a formula of the type

\[
q_s = \delta_b c_b u_b. \tag{6.4}
\]

This point of view is based on the assumption that saltations dominate the motion of the sediment particles. The saltation height follows from the equations of motion for an individual particle; the bed-load concentration was obtained during experiments as a relation of the type

\[
\frac{c_b}{c_o} = a \frac{T}{D_s}, \tag{6.5}
\]

with the volumetric bed-load concentration \( c_b \), the maximum volumetric concentration \( c_o = 0.65 \), a constant \( a \), the dimensionless bed-shear parameter \( T \) and the dimensionless particle diameter \( D_s \). The latter two parameters are calculated as

\[
T = \frac{\tau_b - \tau_{cr}}{\tau_{cr}} \tag{6.6}
\]

and

\[
D_s = d_{50} \left( \frac{g (s - 1) d_{50}^3}{\nu^2} \right)^{3/2}. \tag{6.7}
\]

The bed-load transport rate according to Van Rijn is given as

\[
q_s = 0.053 \sqrt{(s - 1)g d_{50}^3 D_s^{-0.3} T^{-2.1}} \quad \text{if } T < 3 \tag{6.8}
\]

\[
q_s = 0.053 \sqrt{(s - 1)g d_{50}^3 D_s^{-0.3} T^{-1.5}} \quad \text{if } T \geq 3, \tag{6.9}
\]

the second equation was introduced because for higher values of the dimensionless bed-shear parameter, eq. (6.8) was found to overpredict the transport rates. The experiments which yielded this formula took into account median particle diameters \( d_{50} \) up to 2\,mm and Froude numbers smaller than 0.9.

\(^{2}\)According to [67], for nearly uniform the mean particle diameter is about 1.1 to 1.3 times larger than the \( d_{50} \)-parameter of the particle-size distribution curve; furtheron this reference proposes that because of the weak dependence of the transport formula on the particle diameter also the median one, the \( d_{50} \), may be used.
6.1. Sediment Transport Models

6.1.2. Steeper Slopes

SMART and JAEGGI [61] observed during their experiments from 1983 of sediment transport for slopes ranging from 3 to 20% that for the higher slope values the common subdivision in bed-load and suspended-load transport is no longer valid: High transport rates and low relative roughnesses \( h/d \) make a distinction vague because during the experiments saltating particles moved often into regions near the water surface or left the body of water. Furthermore particle interactions took place in form of collisions and rebound due to occurring high transport rates. These processes prevent that a particle sinks to the bottom and evoke a suspended-transport-like movement. The transport mechanism however remains the same as for mild slopes in contrast to the case of debris flow occuring at slopes steeper than 20%. Although, a debris-flow-like behaviour\(^3\) was observed in that water and sediment form a more uniform mixture and large sediment particles which were moved over the whole flow depth. These experiences show the difficulties in formulating transport laws for the steep slope region.

The transport formula developed by SMART and JAEGGI [61] in the course of the mentioned experiments reads

\[
\frac{q_s}{q_{red}} = \frac{4}{s-1} \left( \frac{d_{90}}{d_{30}} \right)^{0.2} \frac{J^{0.61-\theta_{cr}}}{\theta_{m}^{0.14} \theta_{red}^{0.18}},
\]

where \( J \) denotes the slope and \( q_r \) a reduced specific water discharge considering wall drag. This parameter is calculated iteratively as

\[
q_{red} = v_{red} h \left( 1 - 1.41 \sqrt{\frac{q_s}{q_{red}}} \left( \frac{s-1}{g d_{m}^3} \right) J^{1.14} \theta_{red}^{0.18} \right)
\]

with the velocity \( v_{red} \) taking friction effects into account; it is calculated as

\[
v_{red} = 2.5 \sqrt{ghJ \left[ 1 - \exp(-0.05 h/d_{90} \sqrt{J}) \right]} \ln(8.2 h/d_{90}),
\]

This expression was introduced for values of \( 5 \leq h/d_{90} \leq 20 \) because in this range many conventional formulas, e.g. of Keulegan or Manning-Strickler, tend to overpredict the velocities.

Another transport formula was developed by RICKENMANN [54]. He carried out experiments for the same range of steep slopes as SMART and JAEGGI but used clay suspensions of various concentrations as transporting fluid. In doing so, a kind of debris flow was simulated and for hyperconcentrated flows Rickenmann observed that density effects cause an increase in the bed-load transport rates compared to similar clear water conditions.

For the resulting transport formula for turbulent flow behaviour also the data for the clear water tests of SMART and JAEGGI and the low slope clear water data of MEYER-PETER and MUeller were taken into account. It reads

\[
q_s = 3.1 \left( \frac{(s-1)g d_{m}^3}{\sqrt{s-1}} \right)^{1.2} \left( \frac{d_{90}}{d_{30}} \right)^{0.2} \sqrt{\theta_{m} (\theta_{red} - \theta_{cr})} F_{red}^{1.1},
\]

\(^3\)see section 6.2
where $\theta_{\text{red}}$ is calculated as

$$
\theta_m = \frac{h_{\text{red}}J}{(s-1)d_m} \quad (6.14)
$$

with the water height reduced for sidewall influence and including the space occupied by the moving particles. The Froude number $Fr_{\text{red}}$ in eq. (6.13) is calculated as

$$
Fr_{\text{red}} = \frac{u}{\sqrt{gh_{\text{red}}}} \quad (6.15)
$$

and thus evaluated for the reduced water height. This is given by Rickenmann as

$$
h_{\text{red}} = h - 2hB\left(\frac{u}{k_{\text{St}}\sqrt{J}}\right)^{1.5} \quad (6.16)
$$

with the Strickler coefficient $k_{\text{St}}$ and the flume width $B$.

### 6.2. Debris Flows

A sediment transport phenomenon related to steep slopes are debris flows. These are mass movements consisting of concentrated slurries of water, fine particles and larger ones up to rocks and boulders. In contrast to the dilute transport of sediment in water which is to be treated as two-phase flows, debris flows can be considered as one-phase flow, requiring rheological methods in order to model their flow properties.

A critical bed inclination value causing debris flows is about 20%, given sufficiently high water discharges. Another condition for presence of this phenomenon is the supply of un- or not sufficiently consolidated bed material: Deposits, accumulated in a longterm process of continuous erosion, can loose their stability due to intense water discharge and form debris flows. In this case, a slurry is the origin of the debris flow. It can also result from landslides which occur as a consequence of ill-consolidated deposits like moraines or from massive rockfalls; also soils rich of gypsum prone to cause landslides. There are other soil-mechanical relevant factors like increasing loads and increasing pore pressures or water content of the soil which influence the occurrence of a land-slide.

Linked to the water content present in the ground, a further condition for initiation of debris flows is a source of moisture. Apart from present or past rainfalls this can be snowmelt. Cohesive parts of the soil will thus lead to the loss of stability, in conjunction to the high weight of saturated non-cohesive soils.

A point of special interest is the manner of propagation of a debris flow since this depends on the bulk mechanical behaviour of the fluid-solid-mixture which allows a classification. A different aspect for grouping may be the size distribution of involved materials. In general, a characteristic of motion of debris flows are large concentrations of bigger rocks at the leading edge, which besides is often higher than the following parts of the flow. The main body is formed of a rock and mud mixture of high viscosity and rather laminar flowing behavior; within this part of the flow, boulders are transported. The tail shows a decreasing concentration of soil.

\[ \text{(6.14) The dimensionless bed shear stress } \theta \text{ introduced in eq. (B.6) can also be computed as } \theta = \frac{\tau}{\rho g (s-1)d_m} = \frac{hJ}{(s-1)d_m}. \]
6.3. The Bed-Evaluation Equation

Particles. In case of muddy debris flows, a viscoplastic bulk behaviour is observed with rather solid-body-like properties below a critical shear stress value and fluid properties above it. The particle size distribution of these debris flows is rather wide, including cohesive materials which serve as lubrication for contact between coarse particles. This type of particles is lacking in so-called granular debris flows, they exhibit a propagation behaviour which is mainly influenced by frictional and collisional effects between particles, causing dissipation and the need of steeper slopes for continued flowing. A third group of debris flow, the lahar-like, shows at how shear rates a friction-governed bulk behaviour and at higher values viscous properties; this is explained by narrow particle-size distributions with a small amount of cohesive particle fractions.

The motion of debris flows decelerates when the bed inclination decreases or when the flow thins extremely, e.g. due to lateral spreading out of the mixture because of widening channel geometries. A suddenly slowing or stopping leading edge is often overtaken by the main body of the torrent, resulting in the absence of the characteristic front. A further consequence of the loss in speed is the deposition of solids in the channel, forming at the lateral boundaries of the debris flow levees. Muddy debris flows show a rather sudden termination of motion due to low shear rates which do not achieve the critical yield stress value which marks the end of the solid-like behaviour. A slope of 5% is given as critical slope under which the movement may stop. For granular debris flows steeper slopes are required, due to the physical processes of energy losses outlined above; 15% is a guideline. The lowest motion maintaining slopes were experimentally found for lahar-like flows which move over inclinations of less than 1%.

6.3. The Bed-Evaluation Equation

The bed-evaluation equation describes the mass conservation of sediment in a control volume of the ground and accounts thus for morphodynamic processes. Within its formulation

$$\frac{\partial z_B}{\partial t} + \frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} = \frac{\partial \Psi_s}{\partial t} \rho_{Dep},$$  \hspace{1cm} (6.17)

$z_B$ denotes the geodetic height of the ground, $q_s$ the depth-integrated sediment flux or transport rate of particles, $\Psi_s$ the sediment flux between the ground and the water column or the sum of erosion and deposition flux and $\rho_{Dep}$ represents the density of the upper sediment layer. Consideration of this equation yields that the change in time of the geodetic height in a control volume depends on the divergence of the sediment transport rate and the erosion-deposition-source term. If bed-load transport dominates the transport of particles in suspension it can be assumed that the divergence term is much larger than the source term and the homogeneous partial differential equation

$$\frac{\partial z_B}{\partial t} + \frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} = 0$$ \hspace{1cm} (6.18)

results; if suspended-load transport controls the change in bed height, the divergence term can be neglected and the expression

$$\frac{\partial z_B}{\partial t} = \frac{\partial \Psi_s}{\partial t} \rho_{Dep}$$ \hspace{1cm} (6.19)
is obtained from eq. (6.17).
Considering the suspended-load transport necessitates the determination of the sediment transport in the water column in addition.
7. Extension to the CLAWPACK-Library for Dam-Break Problems

The in this work utilized CLAWPACK-library, shortly presented in the following section 7.1, was supplemented by subroutines calculating the flood wave induced by a dam-break for one dimension and computing the caused sediment transport. The chosen system of equations and its approximation are presented in section 7.2, the subsequent section 7.3 overviews the implemented source terms.

7.1. The CLAWPACK-Library

CLAWPACK is a Fortran library for solving, based on Godunov’s method, time-dependent hyperbolic systems of hyperbolic partial differential equations; the name is the abbreviation for Conservation LAWs PACKage. The program is developed at the department of applied mathematics of the university of Washington by the group of Randall LeVeque. The software can be used to solve nonlinear systems of conservation laws, systems of equations including source terms, nonconservative hyperbolic systems and systems with variable coefficients. In this thesis, version 4.3 of CLAWPACK was applied. The basic structure of the for this work relevant parts of the software, providing the solution to one-dimensional problems, is the following:

- A main program preallocates the storage needed for the arrays and calls a subroutine named claw1ez.f, see below.
- The subroutine claw1ez.f reads the initial conditions for the conserved variables and the input data, which contain information about:
  - the desired number of grid cells and the computational domain
  - the number of equations in the hyperbolic system
  - the number of waves in each Riemann solution
  - the initial time step and the maximum time step to be allowed; usage of fixed or variable time steps is possible
  - the desired Courant number and the maximum Courant number to be allowed
  - the order of accuracy of the method: application of a first-order Godunov’s method or of high-resolution correction terms; the choice between different flux limiter functions (no limiter, Minmod, Superbee, van Leer, MC) is possible
Chapter 7. Extension to the CLAWPACK-Library for Dam-Break Problems

- the type of boundary conditions to be applied to each end of the computational domain; the choice between a zero-order extrapolation, periodic, solid wall and user defined boundary conditions is possible
- presence of a source term
- presence of a capacity function in the equation
- presence of auxiliary variables
- the desired number of outputs of the results

For the solution of each time step the subroutine `claw1.f` is called.

- The subroutine `claw1.f` calls the subroutine for the extension of the grid data to the boundary cells, the subroutine `step1.f` for the computation of the new value of the conserved variables after one time step and for a homogeneous system of equation. The call of the subroutine `src1.f` accounts for the influence of the source term to the value of the conserved state variables. This procedure is repeated until the Courant number reaches the desired or the maximum value provided by the user.

- The subroutine `step1.f` computes the solution to the wave propagation algorithm applied to the homogeneous system of hyperbolic partial differential equations: The subroutine `rp1.f` solves the Riemann problem and yields the change in the conserved variables due to the left- and right-going waves, then the update according to Godunov’s method follows. In case of a computation with a high-resolution method and flux limiter functions, the correction fluxes are computed in another subroutine and applied to the Godunov-updated conserved state variables.

7.2. The Considered System of Equations

In order to calculate the Riemann problem of dam-breaks on plain or irregular topographies and to account for sediment transport, a system of equations similar to the one used by CASTRO-DÍAZ [17] was considered in this work. It was chosen for reasons explained beneath in this section and is obtained by coupling the one-dimensional mass continuity equation (2.25) to a shallow water equation nearly identical to eq. (2.31) derived in section 2.3.1. The distinction to the precedent one lies in the definition of the direction of $b$, the coordinate-like vector pointing at a fixed bottom: as the reference level is situated in between the free surface and the bottom, the variable $b$ points in the opposite direction of the coordinate $z$, see fig. 7.1. This can be interpreted as the definition of $\tilde{b} = -b$ as depicted in fig. 7.1. A shallow water equation

$$\frac{\partial u h}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h - \frac{1}{2} g h^2 \right) = g h \frac{\partial \tilde{b}}{\partial x}$$

results, equivalently to the one derived precedently in section 2.3 and applicable to fixed bottom cases.

To allow for the problem of sediment transport, a sediment layer $z_B$ is regarded, see fig. 7.1, and
7.2. The Considered System of Equations

the bed evaluation equation (6.18) for bed-load transport presented in section 6.3. This yields the system

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} &= 0 \\
\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h - \frac{1}{2} gh^2 \right) &= gh \frac{\partial \tilde{b}}{\partial x} - gh \frac{\partial z_B}{\partial x} - gh S_f \\
\frac{\partial z_B}{\partial t} + \frac{\partial q_{sx}}{\partial x} &= 0.
\end{align*}
\] (7.2)

With the definition of a variable \( S = \tilde{b} - z_B \), the right hand side of the momentum equation and the time dependent variable of the bed-evaluation equation can be rewritten: The time derivative of \( S \) reads

\[
\frac{\partial S}{\partial t} = - \frac{\partial z_B}{\partial t},
\] (7.3)

since the height of the fixed bottom, \( \tilde{b} \), does not change in time. Eq. (7.3) yields for system (7.2)

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial uh}{\partial x} &= 0 \\
\frac{\partial uh}{\partial t} + \frac{\partial}{\partial x} \left( u^2 h - \frac{1}{2} gh^2 \right) &= gh \frac{\partial S}{\partial x} - gh S_f \\
\frac{\partial S}{\partial t} + \frac{\partial q_{sx}}{\partial x} &= 0.
\end{align*}
\] (7.4)

These equations can be written in the vector equation notation as an hyperbolic system with a source term and a non-conservative term:

\[
\phi_{,t} + (f(\phi))_{,x} = d(\phi)\phi_{,x} + \psi(\phi),
\] (7.5)
where
\[
\phi = \begin{bmatrix} h \\ hu \\ S \end{bmatrix}, \quad f(\phi) = \begin{bmatrix} hu \\ u^2h + \frac{1}{2}gh^2 \\ 0 \end{bmatrix}, \quad d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & gh \\ 0 & 0 & 0 \end{bmatrix}, \quad \psi(\phi) = \begin{bmatrix} 0 \\ -ghSf \\ -q_{sx,x} \end{bmatrix}.
\]

(7.6)

With the Jacobian matrix \( f'(\phi) \) of \( f(\phi) \)
\[
f'(\phi) = \frac{\partial f(\phi)}{\partial \phi} = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 + gh & 2u & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]
and the definition of a matrix \( A \) as
\[
A(\phi) = f'(\phi) - d(\phi) = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 + gh & 2u & -gh \\ 0 & 0 & 0 \end{bmatrix}, \quad (7.8)
\]

system (7.5) can be written as a non-conservative hyperbolic system:
\[
\phi_t + A(\phi)\phi_x = \psi(\phi). \quad (7.9)
\]

This expression is also referred to as an augmented system, see e.g. [32], since the conserved quantities of the shallow water equations are augmented by the "bathymetry function" \( S^1 \). The introduction of this variable and thus the non-conservative term \( d \) enables the computation of water height and momentum coupled to the bed elevation; the latter influences the classical conserved variables of the shallow water equations directly in every time step. Although, system (7.9) represents not a real unsteady approach where water flow and riverbed are calculated simultaneously and in dependence of each other: This would require that the sediment transport rate, which is in the presented system contained in the source term, is included in the flux vector \( f \). In this case the matrices \( f' \) and \( A \) would read
\[
f'(\phi) = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 + gh & 2u & 0 \\ -q_{sx,h} & -q_{sx,(uh)} & 0 \end{bmatrix}, \quad A(\phi) = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 + gh & 2u & -gh \\ -q_{sx,h} & -q_{sx,(uh)} & 0 \end{bmatrix}. \quad (7.10)
\]

In this form a direct dependence of the water height and the momentum on the sediment flux can be accounted for, additionally to the influence of the bed elevation to these parameters. The disadvantage of the resulting system of eigenvectors and eigenvalues, see [17], is that the Roe

\footnotetext{As already mentioned above, a system similar to the one presented by CASTRO-DÍAZ in [17] is used; this original system was developed in order to compute (tsunami) waves evoked by debris avalanches on the sea ground. So the vector describing the height or rather the depth of the fixed bottom points downwards, seen from some reference level in the water body.}
matrix, enabling a more stable numerical treatment, can only be approximated. Due to this fact, a system with matrix (7.8) will yield more accurate results if the time scale of sediment movement is small compared to the time scale of change in water height and momentum. In this case, the loss of momentum per time step owing to the transport of sediment particles would be negligible. This is one of the assumptions of common practice. Nevertheless, for the case of very intense, debris flow-like sediment transport it seems to be reasonable to simulate dam-breaks based on the completely coupled system (7.10). Because the determination of the eigenvalues, eigenvectors and approximated Roe matrix is very complicated due to the included derivative of the sediment transport formula with respect to both the water height and the momentum, for the calculations done in this work a system of equations was chosen which is akin to the one of Castro-Díaz: This enables a comparatively simple changeover to the utilization of the eigenvalues etc. given in [17].

Several further systems of equations are possible and discussed in e.g. [37].

### 7.2.1. Approximated Solution of the Riemann Problem

The Riemann solution of the considered system (7.9) of equations is approximated through the Roe linearization whose principles are described in section 5.2.1. The Roe matrix $\hat{A}$ to system (7.9) is given by

$$\hat{A} = \begin{bmatrix}
0 & 1 & 0 \\
-\hat{u}^2 + g\hat{h} & 2\hat{u} & -g\hat{h} \\
0 & 0 & 0
\end{bmatrix}$$

with

$$\hat{h} = \frac{1}{2}(h_r + h_l), \quad \hat{u} = \frac{u_r\sqrt{h_r} + u_l\sqrt{h_l}}{\sqrt{h_r} + \sqrt{h_l}}$$

and the eigenvalues

$$\hat{\lambda}_1 = \hat{u} - g\hat{h}, \quad \hat{\lambda}_2 = \hat{u} + g\hat{h}, \quad \hat{\lambda}_3 = 0$$

and eigenvectors

$$\hat{r}_1 = \begin{bmatrix}
1 \\
\hat{u} - \sqrt{g\hat{h}} \\
0
\end{bmatrix}, \quad \hat{r}_2 = \begin{bmatrix}
1 \\
\hat{u} + \sqrt{g\hat{h}} \\
0
\end{bmatrix}, \quad \hat{r}_3 = \begin{bmatrix}
g\hat{h} \\
0 \\
-\hat{u}^2 + g\hat{h}
\end{bmatrix}.$$  

In case of transonic Riemann solution, it is made use of the Harten-Hyman entropy fix presented in section 5.2.2.1. The solution in dry bed-regions is calculated according to the solution presented in section 5.4.2, depth-positivity is in these cases preserved by use of the HLLE solver, see sec. 5.2.3. The latter simplifies for the dry-bed case, included the results of the aforementioned section, to

$$s^1 = \min \left[ 0, u_l - \sqrt{g h_l} \right],$$

$$s^2 = \max \left[ 0, u_l + 2\sqrt{g h_l} \right],$$

(7.15)
for a right-hand dry state, for a left-hand dry state it is
\[ s_1 = \min \left[ 0, u_r - 2\sqrt{gh_r} \right], \]
\[ s_2 = \max \left[ 0, u_r + \sqrt{gh_r} \right]. \]  
(7.16)

### 7.3. Source Terms

#### 7.3.1. Viscosity

GERBEAU and PERTHAME show in [33] a derivation of a viscous Saint-Venant system with the aim of application to dam-break problems. They introduce the viscous term
\[ +4\nu \frac{\partial}{\partial x} \left( h \frac{\partial u}{\partial x} \right) \]  
(7.17)
which has to be added to the right-hand side of the momentum equation. Its derivation is rather long, so is may be refered to [33] for further information.

Expression (7.17) was implemented into a subroutine called by the src1.f-subroutine of the CLAWPACK-library.

#### 7.3.2. Friction

Several approaches to model the friction term \(-ghS_f\) exist, e.g. those suggested by Darcy and Weisbach and by Manning and Strickler. The formulas these authors developed can be regarded as the most widespread and in practical use confirmed models; the one of Darcy and Weisbach was implemented into a subroutine linked to the source term-calculating one src1.f of the CLAWPACK-library.

A problem that is described by e.g. SMART and JAEGGI, as mentioned in section 6.1.2, is that velocities at small water heights are overpredicted by friction formulas. They propose a reduced velocity calculated according to eq. (6.12) if the relative water height \(h/d_{90}\) ranges between 5 and 20. For numerous cases this calculation led to unphysical high velocities, probably partly due to the following circumstance: the wetting front of the propagating wave features nearly diminishing water heights, at least from a physical point of view. As the conserved state variable considering the kinetic energy is the momentum \(hu\), the velocity \(u\) is calculated through the division of momentum and height. This yields in certain cases, i.e. for small values of \(h\), unrealistic results.

That problem is crucial since the velocity is an e.g. sediment transport determining quantity, thus it is of major interest to calculate it realistically.

As a possibility to receive velocities and also front shapes agreeing with the physically based idea, the following procedure was found: the local friction factors are computed according to the Darcy-Weisbach or Manning-Strickler formula; in case of discharges below the limit of 1% of the maximum discharge occurring in the present time step, this last value is maintained as a constant \(S_f\). This procedure leads to satisfying results for example in case of the experimental data of the U.S.Geological Survey [22].
7.3. Source Terms

7.3.3. Sediment Transport

Assuming that bed-load transport dominates the sediment movement under dam-break induced waves in the Alpine regions\(^2\), the models presented in section 6.1 were implemented as source terms.

The consideration of suspended-load transport is a relevant process for regions richer in fine particles and necessitates a system of equations taking into account the advection of the sediment concentration.

\(^2\)see description of the task, page iv
8. Validation of the Implemented Code

This section presents the validation of the extension to the CLAWPACK-library which was carried out with respect to analytical solutions and to experimental results. Section 8.1 considers the case of dam-breaks on horizontal beds, making use of the analytical solution by Ritter, the one by Stoker and experiments published by Martin and Moyce. Section 8.2 presents the validation for dam-breaks on inclined planes with respect to the analytical solution by Dressler and to an experimental study by the U.S. Geological Survey. Results of the validation are summed up in section 8.3.

8.1. Dam-Break on a Horizontal Plane

8.1.1. Comparison with the Analytical Solution by Ritter

In 1892 Ritter [55] gave the first analytical solution to the dam-break wave problem. He considered a channel of horizontal bottom with smooth bed and walls and rectangular cross-section; the dam runs at right angle to bed and breaks instantaneously. The initial conditions are given as

$$h(x,t = 0) = \begin{cases} h_0 & \text{if } x < 0, \\ h = 0 & \text{if } x > 0, \end{cases} \quad (8.1)$$

for the water height and as

$$u(x,t = 0) = 0. \quad (8.2)$$

for the velocity. Thus downstream of the dam the bottom is assumed to be dry and the fluid is initially at rest.

The solution reads

$$h(x,t) = \frac{2\sqrt{gh_0 - x}}{gt} \quad (8.3)$$

$$u(x,t) = \frac{2}{3} \left( \sqrt{gh_0 + \frac{x}{t}} \right) \quad (8.4)$$

it is defined in the interval

$$-1 < \frac{x}{t\sqrt{gh_0}} < 2. \quad (8.5)$$

Outside of this interval the quantities $h$ and $u$ are not affected by the wave and remain hence identical to the initial conditions.

The solution for the wave shows the following properties: Its profile is parabolic with a tangent to the channel bottom at its front; at the dam site the water height is constant in time with
\( h(x = 0, t) = \frac{4}{3} h_0 \). The velocity is linear along the channel with the values Zero at the negative front and \( 2\sqrt{gh_0} \) at the positive front; this quantity, too, is independent from time at the location of the dam where \( u(x = 0, t) = \frac{2}{3} \sqrt{gh_0} \) is hold. At this point the discharge thus follows as constantly \( q(x = 0, t) = \frac{8}{27} h_0 \sqrt{gh_0} \). The propagation of the wave takes place at the velocity \( u(x = \text{front}, t) = 2\sqrt{gh_0} \) in the downstream-direction and upstream at the celerity \( c(x = -\text{front}, t) = \sqrt{gh_0} \). In this region the flow is subcritical with \( u < \sqrt{gh_0} \) whereas upstream it is supercritical with \( u > \sqrt{gh_0} \); at the dam site the flow is critical with the Froude number \( Fr = \frac{u}{\sqrt{gh}} = 1 \).

The results of the Ritter solution for water height, velocity and discharge are shown in the following figures 8.1 to 8.5 as blue dashed lines\(^1\); the black solid lines represent the solution obtained with CLAWPACK. The quantities are plotted for several time steps up to \( t = 10s \), allowing for comparative examination of the development in time. Computations were carried out for an initial water height \( h_0 = 10m \) and grid width \( \Delta x = 0.01m \), gravity was taken into account as \( g = 9.81m/s^2 \).

The values obtained for the water height, depicted in fig. 8.1, agree well for both computations, the constant value of \( \frac{4}{3} h_0 = 4.4m \) at the dam site is hold. At positive and negative front, small differences can be seen in the details of these regions depicting fig. 8.2: due to the numerical diffusion of the by CLAWPACK applied Godunov method, results for the "end values" of the fronts are not calculated exactly. Thus at the negative front the transition from the water in rest to the wave, where the analytical solution is not differentiable, the simulated solution is smoother and also the parabolic shape of the positive front is inaccurate. Here, in the height of 0.05\% of \( h_0 \) or 1\% of the constant height at \( x = 0 \), respectively, the effect of the numerical diffusion vanishes.

Fig. 8.3 shows the velocity. For that parameter, the slopes of the linear part of both curves are nearly identical. Differences are visible in the maximal values: The solution obtained with CLAWPACK is about 5\% smaller than the analytical one. Furtheron, at the extreme wetting front velocities of nearly Zero are computed, resulting from this the front is slowed down compared to the analytical solution. These effects can be explained as follows: the conserved and thus directly computed variables of the simulation with CLAWPACK are the water height \( h \) and the discharge rate \( hu \), thus the velocity is calculated as \( u = hu/h \) a posteriori from the results of the conserved variables. Errors in these latter due to e.g. numerical diffusion will thus appear amplified in the velocity. So results for this quantity will have to be considered cautiously.

The simulated discharge per unit channel width is depicted in fig. 8.4; it agrees in general well with the analytically obtained data. Again, differences can be seen due to inaccuracies caused by the 1st order method: in the wetting front the transition from the moving wave to the dry bed is less smooth and in the area \( x = 0 \) maximal are slightly underestimated by about 0.03\%, see fig. 8.5. This inaccuracy attributes to the "curve" in the envelope of the velocity maxima in the same region.

\(^1\)Unfortunately, the dashed line style is visible only for enlargements of the document of about 250\%, this seems to be a problem of the PGF package.
Figure 8.1.: Analytical solution by Ritter (blue, dashed) and simulated results (black, solid): water height
Figure 8.2: Analytical solution by Ritter (blue, dashed) and simulated results (black, solid): water height, details of negative wave and wetting front.
8.1. Dam-Break on a Horizontal Plane

**Figure 8.3.** Analytical solution by Ritter (blue, dashed) and simulated results (black, solid): velocity

**Figure 8.4.** Analytical solution by Ritter (blue, dashed) and simulated results (black, solid): discharge per unit channel width
Figure 8.5.: Analytical solution by Ritter (blue, dashed) and simulated results (black, solid): discharge per unit channel width, details of negative wave and wetting front
8.1.2. **Comparison with the Analytical Solution by STOKER**

In 1957 STOKER [64] presented an analytical solution to a dam-break flood wave on horizontal, initially wet bed. This leads to the following peculiarity: The parabolic part of the propagating wave ends in a region of uniform flow where water depth and velocity are constant in space and time.

The figures 8.6 to 8.8 show analytical and simulated data for water height, velocity and discharge per unit channel width in dimensionless quantities at the dimensionless time \( t^+ = 2 \). The simulation was carried out with initial water heights \( h_l = 1.0, h_r = 0.01 \), grid width \( \Delta x = 0.01 m \) and CFL number 0.2.

All courses match well except the negative and positive wave fronts where again numerical diffusion influences accuracy.

Figure 8.9 shows as an example the typical course of the Froude number in case of a dam-break flow: The negative wave propagates in a subcritical way, the critical point with \( Fr = 1 \) lies at the dam site at \( x = 0 \) and the positive front is supercritical. In this region a peak is observable, it is a consequence of the numerical diffusion.

![Figure 8.6: Analytical solution by STOKER (blue, dashed) and simulated results (black, solid): water height](image-url)
Figure 8.7: Analytical solution by Stoker (blue, dashed) and simulated results (black, solid): velocity

Figure 8.8: Analytical solution by Stoker (blue, dashed) and simulated results (black, solid): discharge
Figure 8.9.: Froude number for the simulation of the problem of STOKER
8.1.3. Comparison with Experimental Data by MARTIN and MOYCE

The simulated results for computation on a horizontal plane were compared with the experimental data of MARTIN and MOYCE [45] who reported in 1952 their outcomes of a study of the collapse of fluid columns over initially dry bed. An impression of these early experiments is given in fig. 8.10. They were carried out in a Perspex channel which is assumed to be hydraulically smooth and with dry-bed initial conditions. Results are given in non-dimensional parameters. The simulation was carried out with a dimensionless grid width of \( x^+ = 0.014 \) and a CFL number of 0.2.

The authors of the cited publication measured the time at which the flood wave arrived at certain distances from the gate; the data can be seen in fig. 8.11 as dashed lines. The solid lines in this figure represent the numerical results; different water heights (corresponding to 1mm, 1.5mm, 2mm) were used in order to determine the arrival time of the wave: On the one hand it is not clear how sensitive the measuring instruments available at this time were, on the other hand numerical diffusion affects the simulated results and causes a slight retardation of the extreme front of the wave in comparison to the analytical solution. Even though it is to question in what extent an analytical solution applies to a laboratory experiment since e.g. due to friction the
vanishing water heights of the exact solution at the wetting front will not appear in real cases: It has to be taken into consideration that in the simulated results water heights which may be measurable in reality may occur time-shifted. So very rather small values were chosen in order to register the arrival of the wave properly. Each of the three solid lines marking simulated results represents one of the chosen heights; these curves differ negligibly in slope.

At the non-dimensional distance 1.44 of the front, measured from the wall behind the water column, all experimental and simulated times are set to an identical dimensionless time 0.8 as can be observed in the graph. On the first 10 distance units both data match very well, negligible differences are present and probably ascribable to effects of the gate-lifting which influence the experimental wave.

A comparison of the results for the last fifth of the considered distance from the rear wall shows that the experimental wave decelerates in contrast to the simulated one. It is to be assumed that at this advanced state at which a major part of the wave is rather flat, in the experiment friction and thus dissipation play a role, causing a loss in momentum and thus deceleration. These effects are not accounted for in the simulation. As the Perspex channel utilized during the experiments is nearly frictionless, computations considering friction yielded even for very small relative roughnesses results which did not reproduce the initial phase.

Figures 8.12 to 8.14 show water height, velocity and discharge per unit channel width for several simulated dimensionless times. Again the course of the velocity shows in the front region the "cutted" peak; the curve for the first time step shows the real maximal value\(^2\). Due to the linear dependence of the velocity on the distance from the gate, "real" maximal values are also obtainable by extending the linear part of the velocities until it intersects with a line orthogonal to the abscissa, drawn at the wave front. For the calculated case the error is with about 25% rather large. A reduction of the CFL number will yield more accurate results for this quantity; the shape of the velocity curves can be taken as an indicator on reliability of its computed values.

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\(^2\)The initial time step was set to to \(1 \cdot 10^{-5}\) s, in dimensional time.
Chapter 8. Validation of the Implemented Code

**Figure 8.12.** Results of the simulation of the experiment by Martin and Moyce: water height

**Figure 8.13.** Results of the simulation of the experiment by Martin and Moyce: velocity
8.1. Dam-Break on a Horizontal Plane

**Figure 8.14.** Results of the simulation of the experiment by MARTIN and MOYCE: discharge per unit channel width
8.2. Dam-Break on an Inclined Plane

8.2.1. Comparison with the Analytical Solution by DRESSLER

In 1958 DRESSLER [29] reported an exact solution for an unsteady non-linear wave evoked by dam-breaks in a sloping channel on initially dry bed and for a homogeneous system of partial differential equations, thus without friction. The solution is obtained via a Hodograph inversion and a Legendre function; its presented results consider in dimensionless quantities the negative wave in terms of its water heights and velocities at certain times.

The simulation was executed with a dimensionless grid width of \( x^+ = 0.014 \), a CFL number of 0.2 and a water reservoir of initially triangular shape if regarded from the side, compare fig. 8.15. This one and the following up to fig. 8.20 depict analytical and simulated results. Both data match well. Likewise in the horizontal case, differences can be seen at the extreme negative front where the simulated wave shows the numerical diffusion of the used Godunov method. This effect seems to be negligible for the water height, it is restricted to a very small region. In case of the velocity the numerically diffusive behaviour of the method evokes in the wetting front a "cutted peak"; see the discussion on this problem given in section 8.1.1.

Figures 8.21 to 8.23 depict simulated results for water height, velocity and discharge per unit channel width for several time steps, plotted onto a horizontal plane. In general a flattening of the wave as time advances can be seen, an acceleration of the front and an increasement of discharge. In some pictures, especially in the velocity-depicting one, numerical problems at the negative front occur for late times of simulation. In this region the in section 5.4 described numerical treatment of the "dry-bed case" at the left hand side of the reservoir is prone to the calculation of negative water heights, a problem that is described e.g. in [16] and still a topic of research. In fig. 8.22 the effect is clearer visible than in the other graphs, that is explicable as follows: During the simulation all quantities are computed in their physical dimensions, the results were later for visualization non-dimensionalized. In order to do so, the velocity was divided by a value lower than one, leading to an amplification of the numerically caused nonphysical values. For the given time steps and also for later ones, reliability of the results is not affected by the numerical problems.

\[^3\text{The analytical solution is given in a form which still necessitates rather complex mathematical operations in form of e.g. transformations before results utilisable for this purpose are obtained. So the diagrams given in [29] are depicted.}\]
8.2. Dam-Break on an Inclined Plane

Figure 8.15.: Analytical solution by Dressler and simulated results for $t = 0$, $t = 0.1$ and $t = 0.3$; [29]
Figure 8.16. Analytical solution by DRESSLER and simulated results for $t = 0$, $t = 0.1$ and $t = 0.3$; [29]
8.2. Dam-Break on an Inclined Plane

Figure 8.17: Analytical solution by Dressler and simulated results for $t = 0.6$ and $t = 1.0$; [29]
Figure 8.18.: Analytical solution by DRESSLER and simulated results for $t = 0.6$ and $t = 1.0$; [29]
8.2. Dam-Break on an Inclined Plane

Figure 8.19.: Analytical solution by DRESSLER and simulated results for $t = 2$ and $t = 3$; [29]
Chapter 8. Validation of the Implemented Code

(a) Analytical solution; \[29\]

(b) Simulated solution

**Figure 8.20.** Analytical solution by DRESSLER and simulated results for \( t = 2 \) and \( t = 3 \); [29]

**Figure 8.21.** Results of the simulation of the analytical solution by DRESSLER: water height
8.2. Dam-Break on an Inclined Plane

**Figure 8.22.** Results of the simulation of the analytical solution by DRESSLER: velocity

**Figure 8.23.** Results of the simulation of the analytical solution by DRESSLER: discharge per unit channel width
8.2.2. Comparison with an Experimental Study by the U.S. Geological Survey (USGS)

In order to verify the advancement of the wetting front and the applicability of the model to rough beds, an experimental case was simulated which was carried out in 2007 in the USGS debris-flow flume depicted in fig. 8.24. This flume has constant slope of $31^\circ$, corresponding to 60%; for the regarded experiments its bed was tiled with roughness elements of 15mm height. According to Denlinger and O’Connell who used the experiment recently as test for a numerical model and who belong to the same institution, this value is identical to the effective roughness $k_s$, see [22].

The CLAWPACK computation was simulated with grid width $\Delta x = 0.01m$ and a CFL number of 0.1; friction was accounted for with the Darcy-Weisbach model and the roughness length of $k_s = 0.015m$ as explained above. Likewise in the experimental setup the reservoir is of triangular shape, viewed from the side, with the initial water height $h_0 = 1.8m$ and volume $V_0 = 6m^3$. The propagation of the wetting front during the experiments is given in a graph in [22] along with several numerical results obtained by the authors of the same publication; it is depicted in fig. 8.25a. In fig. 8.25b the front propagation of the simulation with CLAWPACK is depicted, the data are nearly identical to the predominant solid black line in the upper graph and thus the experimental results. A slight difference can be seen in the slope which is about 1° steeper in the results simulated with CLAWPACK. This leads to a negligible slower front propagation which may be caused by the numerical diffusion discussed in the precedent sections.

The figures 8.26 to 8.28 display the shapes of water height, velocity and discharge per unit channel width for several time steps of the simulation. At consideration of the water height in the region of the front, small peaks are visible. They originate from the influence of friction in this region where velocities are maximal, causing high values of the friction term which includes the
8.2. Dam-Break on an Inclined Plane

Figure 8.25.: Front propagation in the USGS experiment and the simulation with CLAWPACK
square of the velocity. Thus the front is decelerated by stiction and dammed up. The discharge curves show this effect of the friction in an even more perceivable way.

In the bulk of the fluid and at the left hand side where the reservoir ends, oscillation-like deflections can be seen in the figures, amplifying as simulated time advances. At the left hand side, they results from the same problem in numerical treatment of the "dry-bed case" described in the precedent section. In the bulk of the fluid, roll waves seem to evoke them: The Froude number depicted in fig. 8.29\(^4\) shows values larger than 2. In [7] it is shown that in this case, the oscillation-like movements in the graphs for water height and so on are the result of the mentioned physical phenomenon which is subject of intense research. Also in videos of the experiments roll waves can be seen.

Another possible reason is a numerical one: The numerical solver is developed for homogeneous systems of equations. In this case, a system with the friction source term is used, leading to a non-homogeneous system. In the case that the change per time step in the source term predominates the change in the homogeneous system of equations, the numerical solution will fail since the utilized explicit numerical scheme is prone to amplifications in the predominant term, leading to high-oscillatory solutions. It can be shown that the dominance of the source term depends on the wavelength on the one hand and on the other hand on the considered bed inclination, see e.g. [9]. If for one of both in comparison to the initial water height of the dam large values are present, the source term will dominate the system of equations and according to [43] kinematic wave and steady flow conditions occur. These cannot be resolved adequately with the utilized numerical scheme as explained above.

For longer simulation times than taken into account in the graphs discussed above, wider parts of the wave body are affected by the roll waves and by numerical difficulties due to wavelength and steep slope. This causes that the computation fails. Nevertheless the simulation shows that the program yields with the utilized friction law reliable results which are nearly identical to the experimental ones and that the physical phenomenon of roll waves seems to be reproducible.

\(^4\)In the front, numerical diffusion causes due to extremely small water heights unrealistic large Froude number values.
8.2. Dam-Break on an Inclined Plane

Figure 8.26.: Results of the simulation of the experiment by the USGS: water height

Figure 8.27.: Results of the simulation of the experiment by the USGS: velocity
Chapter 8. Validation of the Implemented Code

**Figure 8.28.** Results of the simulation of the experiment by the USGS: discharge per unit channel width

**Figure 8.29.** Froude number for the simulation of the experiment by the USGS
8.3. Summary of the validation

The elaborated program was validated basing on analytical solutions for horizontal and sloped beds as well as basing on experimental studies for both cases. Friction and non-friction conditions were accounted for.

It was found that the results obtained with CLAWPACK agree very well with the reference data. Numerical diffusion influences results at the extreme wave fronts due to the applied numerical method, being this of first-order accuracy since high-resolution methods such as presented in section 5.3 failed during simulations on sloped beds. Concerning water height and discharge, effects of numerical diffusion seem to be negligible. As in case of the velocity, the at the positive wave front occurring maximum value is affected and underestimated, this quantity should be used with care if it is considered. Concerning the influence of results at the positive wave front, a fact mentioned in section 2.3.2 has to be repeated: In this region, curved streamlines occur in reality. The shallow water equations cannot account for these. So results obtained for this area of the wave and basing on a shallow water-approach will differ from reality even if they are computed exactly.

The implemented Darcy-Weisbach friction model yields results agreeing with experimental findings.

In case of very steep slopes, large wavelengths or a combination of both, development of roll waves is possible and also of kinematic waves. This restricts the computability of such cases to shorter time periods since the numerical method is prone to fail in such cases, see the precedent section 8.2.2.
9. Comparison with Simplified Methods

For calculation of dam-break flood waves, federal offices recommend simplified methods which remove complexities from the original physical processes. Their reliability is questionable as e.g. publication [44] shows.

Section 9.1 overviews simplified methods recommended by Swiss federal offices and gives a comment on consideration of dam-breaks by German authorities. A program called CASTOR, based on one of the simplified methods applied by federal offices, is presented more closely in section 9.2. Section 9.3 examines results obtained in calculation with this program in comparison to those simulated with the extended CLAWPACK-library. A summary of the comparison is given in section 9.4.

9.1. Methods Recommended by Federal Offices

In 2002 the former Swiss Federal Office for Water and Geology (Bundesamt für Wasser und Geologie, BWG) published document [12], defining criteria for legal regulation of dams. In this context, the term of the "particular danger", concerning the endangering of the downstream riparians through the possibility of dam failure, plays a major role. In order to appraise that particular danger of a dam, by a flood wave potentially affected buildings and institutions must be determined and hence the characteristic parameters of a wave in terms of e.g. its discharge, water height and velocity. In the appendix of the same document [12], two calculation-by-hand methods are proposed, one according to BEFFA\textsuperscript{1} and another one based on a proposition presented by CTGREF\textsuperscript{2}. This latter method exists also as a Java-written program called CASTOR\textsuperscript{3}. Furtheron, in [12] it is refered to [38], a publication of the International Commission On Large Dams (ICOLD) containing an overview of existing approaches of dam-break calculation; this work lists also the previously named ones.

All these three methods can be estimated as simple regarding the considered physical or mathematical complexity, the way of calculation and the detail of outcomes. The two hand-calculation approaches enable to compute for a given point downstream of the dam the maximum occurring values of water height, velocity and discharge, making use of several tables and diagrams. These restrict the methods to certain roughness factors and bed slopes and limit accuracy. Temporal resolution of incidents, i.e. the propagation of the wetting front, is not obtainable but, in case of the CTGREF method, arrival times of the wave front can be calculated. Furthermore, this method takes the breach form into consideration. With the BEFFA-method, lateral propagation


\textsuperscript{2}Centre technique du génie rural des eaux et des forêts. Appréciation globale des difficultés et des risques entraînés par la construction des barrages, note technique No 5.

\textsuperscript{3}French for beaver
is a possible result. A basis of both computation models is the initial reservoir volume. For
detailed information about the calculation procedure it is referred to the cited publications.
In this work, attention is focused to a comparison with the program CASTOR: It is one of the
methods recommended by Swiss federal offices and used in practice, as well as the calculation-
by-hand ones, but in opposite to these it seems as a computer-based calculation method more
evenly matched to simulations with CLAWPACK. The program is presented in the following
section 9.2.
To get a more complete picture of the actual handling of the dam-break problem by parts of
the authorities, it was investigated which course of action for design of dams German norms
stipulate. The norm DIN 19700 "Talsperren" [23] from 2004 is the binding corpus of legislation
concerning the requirements for dam plants, it consists of six parts of which numbers DIN 19700-
11 to DIN 19700-15 treat, amongst other topics, the potential of endangering. The flood wave
occurring if the dam fails is not considered; attention is focused on flood protection during normal
operation: Evidently dams are regulated regardless of their location. The volume is taken into
consideration, as is in Switzerland. It is to assume that the unequal role of the local difference
in the endangering potential is the result of the different predominant types of dams in both
countries: in Germany nearly exclusively large dams exist, serving for energy or drinking-water
supply, whereas in Switzerland additionally a lot of small reservoirs can be found which were
build in order to provide water for artificial snow in skiing areas. Thus impounding reservoirs of
small volume in sparsely or not at all populated regions prevail here and from a regulatory point
of view it makes sense to distinguish dams based on the potentially by a flood wave affected
commodity values.

9.2. The program CASTOR

CASTOR is an extension of the CTGREF method, developed by CEMAGREF\(^4\). It is written in
Java and obtainable as executable .jar-file, thus no detailed insights into the calculations are
possible.
Input parameters are the geometries of different cross sections downstream of the dam, of the
dam itself and, if desired, of a breach; for these data, roughness factors and slopes can be defined.
Furthermore, a reservoir is to be defined, consisting of length, water elevation and volume;
guarded properties of this input parameter are incident floods entering the reservoir and initial
river discharges, i.e. discharges in a river before the dam breaching. Additional potential factors
are hydraulic structures like weirs or orifices. For the calculation, choice between several sce-
narios is possible: instantaneous or progressive breaching and step-by-step or independent wave
propagation time, where "independent" signifies that only the dam and the calculation section
are considered. According to [49], computation is based on the following steps:

- Estimation of the peak discharge at the dam depending on the progressive of instantaneous
  mode of failure

\(^4\)Centre National du Machinisme Agricole, du Génie Rural, des Eaux et des Forêts
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

• Creation of a dimensionless graph of the peak discharge at the dam and sections in different distances from it; it bases on "previous experience on dam-break waves" [49, p.427]. Influencing factors are, besides the distance of the section, the bottom slope, the Manning coefficient and the volume of the reservoir.

• Computation of the peak water height at a section, resulting from the peak discharge and the geometry of the section and making use of the uniform-flow equation.

• Calculation of the peak velocity, based on the velocity obtained by the uniform-flow equation and a constant correction factor of 1.2.

• Determination of the propagation time from an cross-sectional average of the peak velocities.

In [49] it is stated that 90% of the results obtained with the simplified CASTOR method differ less than 30% for peak water depth and less than 50% for peak discharge, velocity and arrival time from data computed by the Saint Venant equations.

9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Basing on the results of the program validation in section 8 it can be assumed that the implemented code yields reliable and physical results. This section presents results of several test cases which were computed with both the simplified method program CASTOR and with the elaborated program. Dam-breaks on inclined as well as on horizontal beds were simulated, taking into account friction and non-friction conditions. The latter may be irrelevant for application to reality but any calculation method for water flows should be applicable to these cases, too.

Exemplarily a case of 5% bed inclination with friction was calculated with both methods over a longer distance, results are presented in section 9.3.1. As examples for dam-breaks under steep-slope conditions, section 9.3.2 considers a bed inclination of 30% and section 9.3.3 as an example for horizontal beds of a slope of 0%. The latter two cases are calculated for a shorter distance from the dam-site and account for solutions of simulations with and without friction.

All results obtained with CASTOR are computed for a channel of 1 m width in order to ensure comparability. They include besides a "normal" solution one higher-limit and one lower-limit variant. The exact way of computation of these limits is not reported in the available publications about the program; they are derived statistically. All results are considered as peak values, thus they are compared with the envelope of the CLAWPACK solution.

The propagation times of the wave which normally should be part of the solution were not calculated by the program.

9.3.1. Dam-Break on a Bed of 5% Inclination, with Friction

This section considers dam-break flow on a bed inclined by 5% with Manning-Strickler roughness \( k_{St} = 0.6 \text{ m}^{1/3}/s \), corresponding to a roughness length of 400 mm, [58], and riverbeds charac-
Chapter 9. Comparison with Simplified Methods

terised by boulders and irregularities.
The CLAWPACK simulation was computed with a grid width of $\Delta x = 0.3 m$ and a CFL number of 0.2; for the calculation carried out with CASTOR about 80 rectangular cross sections were defined. Near the dam site spacings of 0.1 m were chosen, enlarging with increasing distance from this location to 50 m. This may represent a typical case of application of the simplified method. The course of the resulting curves show steps; as they appear independently from the distance between cross sections this cannot be the reason for this distinction to reality. Probably the application of graphs, see point 2 in the previous section, causes them.

A water reservoir of $1000 m^3$ Volume and 10 m dam height was considered; for the CLAWPACK simulation it was defined by a triangular cross section as in the cases presented in section 8.2. The following figures 9.1 to 9.3 depict the computed results for a length of 2 km in case of the calculation with CASTOR and 1.5 km in case of the CLAWPACK simulation. For longer distances it was not sure if reliability of the solution is still given because of development of numerical instabilities or roll waves, see discussion in section 8.2.2. The Castor solution is constant at this distance from the dam-site.

The graphs of the in fig. 9.1 shown water height differ visibly. The simulation with CLAWPACK yields decreasing values reaching soon about 1 m whereas the simplified method calculates increasing water heights with a maximum of 40 m for the "normal" and 55 m for the higher-limit solution. This is clearly unphysical, in particular regarding the initial water height of 10 m. This value is calculated at a distance of 750 m from the dam site, the further course of the graph is decreasing until a constant height of 22 m is reached; the lower limit is given as 15 m, the higher as 28 m. These results are clearly larger than the initial dam height, too, and exceed the CLAWPACK-solution by 2000%. Neither shape nor values can be considered as physical.

The velocities, see fig. 9.2 are in contrast to the water heights smaller than the reference solution of CLAWPACK with a difference of constantly about 450%. The shape of the graphs is similar. The in fig. 9.3 depicted discharges are again overestimated by CASTOR with a factor of 4, the graph is continuously decreasing, similar to the one produced by CLAWPACK.

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5Especially since definition of cross sections is not very comfortable in this program but rather time-consuming,
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.1.: Results for a dam-break on an inclined plane: water height

Figure 9.2.: Results for a dam-break on an inclined plane: velocity
Figure 9.3.: Results for a dam-break on an inclined plane: discharge per unit channel width
9.3.2. Dam-Break on a Bed of 30% Inclination

As an example for dam-breaks on steep slopes two simulations on a plane bed of 30% inclination were computed: one without friction and another one with the same friction parameters as described above, i.e. a Manning-Strickler roughness \(k_{St} = 0.6 \text{ m}^{1/3}/\text{s}\) or roughness length of 400 mm. The hydraulically smooth case might not be of great relevance for application to reality but a method calculating water flows should have the ability to yield also for diminishing roughness parameters reliable results.

Computations were carried out with the same parameters as described above; the reservoir volume was about 150 m\(^3\) in order to examine reliability of outcomes obtained with CASTOR for small reservoirs.

The extension to the CLAWPACK-library was verified for similar cases, i.e. the analytical solution by Dressler and the experiments in the USGS-flume.

9.3.2.1. Case without Friction

Results for a computation of a dam-break on steep slopes and hydraulically smooth bed are presented in figures 9.4 to 9.6 in form of graphs for water height, velocity and discharge per unit channel width. The solution of the simplified method shows again no continuous course.

Values for the water height differ by a factor of about 2 with larger results through CASTOR. The courses of the velocity graphs differ considerably: The calculation with the extended CLAWPACK-library yields an increase whereas the simplified method gives nearly constant, slightly decreasing values. The latter are up to 750% smaller than the reference solution; basing on the validation of the program presented in section 8 it can be assumed that these results are well-grounded.

A comparison of the outcomes for the discharge shows accordance.
Chapter 9. Comparison with Simplified Methods

Figure 9.4.: Results for a dam-break on an inclined plane: water height; black - CLAWPACK, blue - Castor

Figure 9.5.: Results for a dam-break on an inclined plane: velocity; black - CLAWPACK, blue - Castor
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.6: Results for a dam-break on an inclined plane: momentum; black - CLAWPACK, blue - Castor
9.3.2.2. Case with Friction

Graphs water height, velocity and discharge as outcomes of both computation methods are shown in figures 9.7 to 9.9.
Similarly to the in section 9.3.1 presented results the graph for the water height increases up to values of more than 70 m with respect to the higher limit case. Regarding the dam height of 10 m this is again not realistic; the difference to the solution of CLAWPACK is a factor of about 20, yielding differences in the order of magnitude.
In contrast to results for this parameter, the velocities calculated with the simplified method are considerably smaller than the reference values. Also in this case due to a factor of 13 a difference in the order of magnitude can be seen.
Outcomes for the discharge agree well for the regarded region but results obtained with CLAWPACK seem to decrease faster, thus computations over larger regions are necessary for further investigation.

![Figure 9.7: Results for a dam-break on an inclined plane: water height; black - CLAWPACK, blue - Castor](image)

Figure 9.7.: Results for a dam-break on an inclined plane: water height; black - CLAWPACK, blue - Castor
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.8.: Results for a dam-break on an inclined plane: velocity; black - CLAWPACK, blue - Castor

Figure 9.9.: Results for a dam-break on an inclined plane: momentum; black - CLAWPACK, blue - Castor
9.3.3. Dam-Break on a Horizontal Plane

Results of both programs were compared for a calculation of dam-breaks on horizontal planes, again for both frictional and non-frictional cases. Computations were carried out with the same resolutions and further parameters as described above; results were considered for the first 400 m downstream of the dam. The initial water height at the dam is again 10 m, the volume was chosen to 100 m$^3$ in order to examine reliability of outcomes obtained with CASTOR for small reservoirs. For a very similar case, the written program was verified with the analytical solution presented by Ritter and the experiments carried out by Martin and Moyce.

9.3.3.1. Case without Friction

This section considers a calculation for a dam-break on a horizontal hydraulically smooth plane. The upper parts of figures 9.10 to 9.12 present results for the case described above, considering, as before, water height, velocity and discharge per unit channel width. All values obtained with the simplified method are 250 to 500% below those calculated with CLAWPACK. As a test, one of the initial conditions defining parameters was changed: CASTOR proposes, depending on the type of rupture, an initial discharge based on which the calculation is carried out. As the user has to insert that value a field, it is possible to modify this number, which was done by increasing the initial discharge by 400%. The results of this computation are pictured in the lower parts of the same figures 9.10 to 9.12. Although values are still too small, the increase leads to much more similar results, apart from consequential too high discharges in the beginning phase of the dam-break obtained with the simplified method. Perhaps a reason for distinctions in outcomes can be found in the calculation of the proposed initial discharge.
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

(a) Both calculations with the same initial reservoir volume

(b) Reservoir volume for the calculation with CASTOR increased by 400%

Figure 9.10.: Results for a dam-break on a horizontal plane: water height
Figure 9.11.: Results for a dam-break on a horizontal plane: velocity
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.12: Results for a dam-break on a horizontal plane: discharge per unit channel width

(a) Both calculations with the same initial reservoir volume

(b) Reservoir volume for the calculation with CASTOR increased by 400%
9.3.3.2. Case with Friction

The test case presented in this section was calculated with the same parameters as before, this time taking friction into account. As in the other frictional computations, a Manning-Strickler roughness of $k_{St} = 0.6 \, m^{1/3}/s$ or a roughness length of 400 mm was used.

Again computations with both the initial discharge proposed by CASTOR and the modified value were carried out. Results are shown in figures 9.13 to 9.15. Similar to the simulation on hydraulically smooth bed the outcomes for the initial discharge proposed by CASTOR are below those computed with CLAWPACK, the difference is between 400 and 600%. Concerning the water height it is difficult to make a clear statement: The values are in the same range but the course of the graph computed by the simplified method is constant over the simulated length, except the last 50 m where a slight decrease can be seen. The reference solution shows this behaviour more distinctly; this agrees better with physically based expectations.

For the increased initial discharge, the water height from the CASTOR simulation are constant, which imposes again the question if physical results are obtained, and 500% above those given by CLAWPACK. The results for the velocity differ by 400% with again too low velocities yielded by the simplified method. The graphs for the discharge show the same values.
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.13: Results for a dam-break on a horizontal plane, calculation with friction: water height
Figure 9.14.: Results for a dam-break on a horizontal plane, calculation with friction: velocity

(a) Both calculations with the same initial reservoir volume

(b) Reservoir volume for the calculation with CASTOR increased by 400%
9.3. Comparison of Results Obtained with CASTOR and CLAWPACK

Figure 9.15: Results for a dam-break on a horizontal plane, calculation with friction: discharge per unit channel width

(a) Both calculations with the same initial reservoir volume

(b) Reservoir volume for the calculation with CASTOR increased by 400%
9.4. Summary of the Comparison

A comparison between results computed with the program Castor, basing on simplified methods, and with the extended CLAWPACK-library was drawn. Cases of different slopes (0%, 5%, 30%) were considered for smooth as well as for friction conditions.

Considerable differences in the obtained values regarding water height, velocity and discharge occur, also in the order of magnitude. The courses of the graphs differ, too; the solutions of the simplified method contain steps and are partly unphysical.

Differences in the values calculated with Castor with respect to those simulated by CLAWPACK are presented in the following table 9.1, differences in the shapes of the curves in table 9.2.

<table>
<thead>
<tr>
<th>friction</th>
<th>5% slope</th>
<th>30% slope</th>
<th>0% slope</th>
<th>30% slope</th>
<th>0% slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>water height</td>
<td>+2000%</td>
<td>+2000%</td>
<td>agrees?</td>
<td>+200%</td>
<td>-300%</td>
</tr>
<tr>
<td>velocity</td>
<td>-450%</td>
<td>-1300%</td>
<td>-600%</td>
<td>-750%</td>
<td>-250%</td>
</tr>
<tr>
<td>discharge</td>
<td>+400%</td>
<td>agrees</td>
<td>+400%</td>
<td>agrees</td>
<td>-500%</td>
</tr>
</tbody>
</table>

Table 9.1.: Differences in the values calculated by Castor with respect to those by CLAWPACK

<table>
<thead>
<tr>
<th>friction</th>
<th>5% slope</th>
<th>30% slope</th>
<th>0% slope</th>
<th>30% slope</th>
<th>0% slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>water height</td>
<td>unphys.</td>
<td>unphys.</td>
<td>differs</td>
<td>agrees</td>
<td>agrees</td>
</tr>
<tr>
<td>velocity</td>
<td>agrees</td>
<td>differs</td>
<td>agrees</td>
<td>unphys.</td>
<td>agrees</td>
</tr>
<tr>
<td>discharge</td>
<td>agrees</td>
<td>agrees</td>
<td>agrees</td>
<td>agrees</td>
<td>(agrees)</td>
</tr>
</tbody>
</table>

Table 9.2.: Shapes of the graphs calculated by Castor with respect to those by CLAWPACK

In the cases of sloped bed, the water height is overestimated by the simplified method, partially by one order of magnitude. For the horizontal bed, the values agree or are underestimated. Regarding the results of Castor, the courses of the graphs are not physical for the frictional, sloped cases and differ in the horizontal case with friction from those obtained with the reference program. For smooth conditions the shapes agree.

All velocity values are underpredicted by the simplified method, in all cases but one by more than 500%, in one with a change of order in magnitude. Considering this quantity it has to be kept in mind that due to numerical diffusion, provided by the utilized Godunov method, velocity values are rather underestimated by CLAWPACK. Thus larger discrepancies are possible. The shapes of the graphs agree for the horizontal and comparatively mild sloped case, for the 30% bed inclination case the simplified method computes constant values for the case without friction which is considered as contradicting reality; in the case with friction the decresement of velocity seems to be to less intense than in the CLAWPACK solution.
9.4. Summary of the Comparison

The discharges are calculated with too high values for the 5\% inclination case; in case of the steep slope results agree, for the horizontal bed they are computed extremely small. The courses of the graphs agree.

Basing on the validation it is assumed that CLAWPACK yields reliable results. Maybe the simplified method is not applicable to small reservoir volumes; also certain slopes are not with physical results computable. It seems questionable if basing on the information provided by the method for cases like the considered ones decisions of importance for safety for general public can and should be taken.
10. Simulation of Sediment Transport under Dam-Break Flow

The simulation of sediment transport under dam-break flow is a topic of actual research and in recent years, several studies were published concerning bed-load transport (e.g. [14], [46], [57]) and suspended load transport (e.g. [13], [47], [51], [70]) caused by dam-break flood waves. It is still a topic that necessitates further investigations regarding both experimental and numerical work.

Section 10.1 presents results of a simulation of experiments carried out by Spinewine and Zech; section 10.2 discusses applicability of actually existing bed-load transport models on dam-break flows.

10.1. Simulation of the Experiments by SPINEWINE and ZECH

In 2007, Spinewine and Zech published an experimental study on sediment transport under dam-break waves on movable beds, [62]. They examined for different initial conditions sediment movement for two types of particles, amongst these an uniform coarse sand with particle sizes ranging from 1.2 to 2.4 mm with \( d_{50} = 1.82 \) mm and density \( \rho_s = 2683 \) kg/m\(^3\); the Manning-roughness factor was estimated as \( k_{st} = 60.60 \) m\(^{1/3}\)/s. A dam-break wave was induced by sudden removal of a gate and over a channel length of 3 m the development in time of water height and sediment movement was observed in differentiating between three flow regions: a pure water layer, a moving bottom layer where sediment transport occurs and a fixed bottom layer, i.e. the motionless sediment bed.

One of the experiments carried out by the cited authors was simulated as a test case for computability of sediment transport under dam-break flow. The considered setup consists of a flat bed of the described sand and a water column of 35 cm height, see fig. 10.1.

![Figure 10.1: Experimental setup of the sediment transport test case, [62]](image)

The following figures 10.2 to 10.6 depict, partly in comparison to the available experimental data, results of the simulation; the latter was carried out with the Van Rijn transport model with the particle properties given above, a grid width of \( \Delta x = 0.016 \) and a CFL number of 0.05. This allows but small time steps and enables thus a more stable simulation since computation of
sediment transport was found to cause numerical instabilities. This difficulty will be discussed in the following section.

Fig. 10.2 shows the propagation of the wetting front for experimental and simulated data. Results are nearly identical, except the beginning of the dam-break. This is not surprising since this state is during experiments prone to be affected by the process of gate-lifting or -lowering and the numerical solution will not be able to take into account effects of turbulence and dissipation properly.

In figure 10.3 simulated and measured wave fronts at different times can be seen. Both shape and height values are simulated very similar to the real case data.

Simulated results for velocity and discharge per unit channel width are visible in fig. 10.4 and 10.5. The velocity maximal values seem to be affected by sediment transport, causing a decrease of momentum and thus lower velocity values, on the one hand and by numerical viscosity on the other hand. The in the wave front region nearly orthogonal to the abscissa running shape of the discharge curve shows the loss in momentum of the fluid due to sediment transport, too.

Fig. 10.6a presents simulated results of the bed height for several time steps. The shape is reasonable: At the wave front sediment is accumulated, in the region of the bulk of the fluid erosion is visible. The negative peak at the location of the gate \((x = 0)\) is explainable by the lack of a program part taking into account the friction angle of fluid and sliding at steep bed gradients; the positive peak at the wetting front is caused by high velocities and small water heights at the same time. The average deposition height is about \(1 \cdot 10^{-3} \, m\), the average erosion depth \(-1 \cdot 10^{-3} \, m\). Fig. 10.6b shows experimental results in form of the fixed bottom layer and the moving sediment layer at \(t = 2s\). A direct comparison between these and the simulated data is not possible since between the two layers measured during the experiments sediment particles in the state of transportation exist; this is not reflected by the utilized bed-evaluation equation.

Thus consideration of the measured heights is not unreservedly reliable. The shape of the experimental layer heights is similar to the simulated bed height and shows the same characteristics. Height values differ at one order of magnitude, but it should be kept in mind that particles in the measured "sediment transport layer"\(^1\) are distributed more dilute and thus more space requiring as if deposited.

At all, effects of sediment transport on fluid and flow regime is simulated in accordance to experimental findings and the application of the transport model by Van Rijn yields physical bed forms whereas too low transport rates seem to be calculated. This is discussed in the following section.

\(^1\)It is to note that this sediment transport layer is much larger than the 3 particle diameters in which in most sediment transport models bed-load transport is assumed to take place.
Figure 10.2.: Results for sediment transport under dam-break flow: wave propagation, comparison with experimental data.
Figure 10.3.: Results for sediment transport under dam-break flow: water height, comparison with experimental data
10.1. Simulation of the Experiments by SPINEWINE and ZECH

Figure 10.4.: Results for sediment transport under dam-break flow: velocity

Figure 10.5.: Results for sediment transport under dam-break flow: discharge per unit channel width
Figure 10.6: Results for sediment transport under dam-break flow: bed height, comparison with experimental data
10.2. Discussion on Applicability of Bed-Load Transport Models

Concerning the usage of sediment transport formulas, in general it has to be kept in mind that they are empirical findings and base predominantly on laboratory experiments which cannot reflect reality exactly. Compared to the large number of physical effects which influence sediment transport, the formulas are based on rather few parameters. Furthermore, experimental possibilities are limited regarding e.g. particle size or flow regime. So even application of the formulas to sediment transport in rivers for which they were developed does not assure that all relevant physical effects are taken into account nor that to a high degree reliable results are obtained.

In case of usage of transport models to dam-breaks these problems are heightened: The flow regime is highly unsteady, large supercritical Froude numbers occur, especially in the wave front. These flow regimes differ considerably from those for which transport models were developed. Comparable it is known that computation of sediment movement in regions of non-steady and non-uniform flow near structures like bridge piers or abutments is still a challenging task. In order to elucidate flow conditions present in the simulation or experiment considered in the preceding section, fig. 10.7a shows the Froude number for several time steps. The graph was cutted at $Fr = 8$ since in the extremal wave front much larger values occur, probably due to the computed extremely small water heights; they are not of physical interest, especially since in addition in this region the shallow water equations cannot account for curved stream lines which occur in reality, see section 2.3.2. The complete positive wave shows highly supercritical flow conditions.

In order to replenish the impression of the flow conditions present in this simulation, fig. 10.7b shows the Reynolds number for the same time steps. They show a turbulent flow regime. It is to note that it is assumed that the averaged velocity profile which is computed with the shallow water equations is not changed by the turbulence. Furthermore, utilization of friction models like the Darcy-Weisbach one is a kind of turbulence modelling.

A further important distinction of dam-break flows to those in rivers is that at the front very low water heights are present. Additionally to the consequences of calculations with low water heights for which unphysical discharge rates result, the fact that in the potential flood wave bed often no rivers are initially present reduces applicability of models since these are developed for transport of water saturated sediments; especially computations for regions rich in cohesive soils may be affected by this problem.

Another difficulty is the particle type: Due to the fact that most water reservoir dams are located in mountainous regions, extremely large particle sizes will be found which are not taken into account in actually available transport models and of which furthermore the effect on fluid and momentum may not be computable by generalizing formulas.

Due to these circumstances, computation of sediment transport under dam-break flow actually is only limitedly possible. Besides the physical reliability of results, also numerical restrictions occur. For the simulation of the above mentioned simulation, the explicit Godunov method necessitated very small time steps. The method by Van Rijn used here proved to be the most stable; tests with the formula of Meyer-Peter and Mueller failed after a few simulated seconds and gave rise to oscillations even if applied at extremely small time steps. Application of the model by Rickenmann evoked unphysical bed forms containing a large jump at the dam site.
The considered transport models were not in a satisfying way applicable to dam-break flows. Given the discussion above, it is questionable if currently transport models exist which can be applied to them and it seems that development of sediment transport formulas for the case of dam-breaks is necessary.

Figure 10.7.: Results for sediment transport under dam-break flow: Froude number and Reynolds number
11. Future Works

The in the given period of time accomplished works and obtained results can be replenished as follows:

- With regard to the implemented extension to the CLAWPACK-library, future works listed hereinafter are thinkable:
  - The actual version of the program calculates one-dimensional flood waves. A solution to the Riemann problem in two and three space dimensions will enable simulation of dam-breaks over real terrain, i.e. curved and discontinuous bed geometries and irregular cross-sections. In the current state of the program it is questionable if application to these inevitably occurring terrains leads to physically reliable results. In the CLAWPACK-library, the possibility to solve Riemann problems in two dimensions is given; the considered system of equations will have to be enlarged appropriately. Dam-break cases which could be simulated are the ones of e.g. Malpasset, the Glacier du Mont Miné, and the Lake Ha!Ha!.
  - In the calculations presented in this work, instantaneous dam-breaching over the whole length is assumed. This scenario is equivalent to the worst-case, causing the highest values for water height, velocity and discharge. For concrete dams this assumption is realistic whereas for some dam types this kind of failure will hardly occur. They form during a continuously process breaches and both breach shape and breaching process will influence the flood wave. The consideration of these parameters will thus be a step to simulations closer to reality.
  - The implemented extension to CLAWPACK enables computation of dam-breaks on dry beds. For these initial conditions, recently methods were described, that enable to calculate the wetting front more stable than the utilized one, see [16]. These could be added to the code.
  - All results presented in the sections 8 to 10 are computed with first order accuracy. Even though they show satisfying accordance to analytical and experimental data, it is desirable to obtain less diffusive results and thus to have more accurate methods at disposal. The CLAWPACK-library enables the application of flux limiter functions which extend the Godunov method to a high-resolution method, see section 5.3. Unfortunately they did not work properly for supercritical flows over inclined beds. Thus adaption of these limiter functions for the considered circumstances will yield to an improvement of numerical results.
  - A subject of actual research is the development and simulation of roll waves. Here further work is possible in order to stabilize the numerical computation and to ensure
physically reliable results. Close to this, also for long wavelengths or very steep slopes, causing development of a kinematic wave, the program can be stabilized.

- With the aim to appraise reliability of simplified methods recommended and utilized by federal offices, future works as presented above are imaginable:
  - Further computations could be carried out, taking into account
    * longer distances downstream of the dam site.
    * different reservoir volumes and initial water heights, also larger values than those considered in section 9.3 but still for rather small dams: They are the ones which are actually not well-founded considered during the authorisation process and impose thus potential risk for the general public.
    * reference results obtained with further programs, e.g. BASEMENT, developed at the ETH of Zurich, [2].
    * further simplified methods; apart from the calculation-by-hand ones mentioned in section 9.1 this should include predominantly further programmed methods: Obtaining results in order to take decisions of great responsibility based on usage of pocket calculators is not very close to technical possibilities and progress should be made in another direction. It should be kept in mind, of course, that those very simple methods can yield much more faster, without great preliminary work, results enabling evaluation of the general kind of situation.
    * different, characteristic initial and boundary conditions concerning geometry of cross-sections, dam-breaching process and breach forms - this is at the current state of the reference program CLAWPACK not yet possible and evokes further work on it, see points listed above.
    * more dimensions in space - in order to do so, the implemented code has to be extended, see above.
    * computations for real dam-break cases as far as geometry data of the state before the incident are available - this will necessitate further work on CLAWPACK, too, see above.
  - Concerning the analysis of results,
    * methods of statistics can be applied in order to obtain statements of more generalized validity.
    * it is to evaluate, in which distances of the dam site parameters of the flood wave are of importance for the so-called "particular danger", see section 9.1; for the considered small dams supplying e.g. ski resorts with water for artificial snow this will depend in general on the season but also considerably local dependencies are to be expected.
In connection with the predominantly large differences of results obtained by application of and CLAWPACK and assuming that the outcomes of CLAWPACK are more reliable, it should be investigated why CASTOR is not very well applicable to the considered cases. This seems to fall in the area of responsibility of the developing institution. Since CASTOR aims and is validated for application to dam-break flows, possibly the application to small reservoir volumes causes poor results. It is thinkable that development of new methods is necessary.

- Concerning simulation of sediment transport, the following measures are possible:
  - The presented results base on a system of equations given in eq. (7.9) with the matrix \( A \) defined in eq. (7.8), yielding an approach with the computation of momentum and water height coupled to the bed elevation. As mentioned in section 7.2, the utilized system of equations can comparatively easily be changed to enable a coupling "in both ways", i.e. the simultaneous computation of water flow and riverbed in dependence of each other. This requires to include the sediment transport rate in the flux vector \( f \) and thus the matrix \( A \), see the cited section.
  - The consideration of both bed-load transport and suspended-load transport is a further step allowing for simulations closer to reality. To account for concentrations of suspended particles, a further conserved state variable is to be added to the system of equations. In dependence of the local concentration its effects on e.g. fluid-sediment mixture viscosity and the relative density could be taken into account; this is of relevance since it influences bed-load transport, see section 6.1.2 and [54].
    A first, simpler possibility is to include simple total load transport models.
  - In order to be in a position to compute sediment transport with the parameters for e.g. size range and density present in nature, this means without adapting them until the desired results are obtained, it seems to be advisable to have transport models at ones disposal which are more suitable to the flow conditions of dam-breaks and actually do not exist. Thus an aspect of future research should be the development of those transport formulas; this affects in the first instance experimental work.
  - Furthermore, effects like bed forms, gravity-induced particle motions, more-dimensional transport and roll waves represent bed-evaluation influencing factors of which consideration will enable results closer to reality.
12. Conclusion

In order to compute the dam-break induced one-dimensional flood wave on arbitrary bed geometries, a Fortran-library called CLAWPACK, developed at the University of Washington, was extended. The imposing Riemann problem is solved, basing on the Godunov method, with first-order accuracy. In doing so, a so-called augmented system, resulting from the Roe linearization is utilized which enables computation of water height and momentum in direct dependency on the bed geometry. Source terms account for viscosity and friction and determine sediment transport in form of bed-load as well as the resulting changes in the bed geometry.

The validation of the elaborated program was carried out based on analytical solutions which account for the non-friction dam-break case for horizontal and sloped beds as well as basing on experimental studies for both geometry situations and additionally for friction conditions. Good agreement of simulated results with the reference data was found concerning water height, velocity, discharge and wave propagation.

The course in space of these parameters is slightly influenced by numerical diffusion caused by the utilized numerical method. The effect on results for water height and discharge is marginal and locally restricted to the extremal positive and negative wave fronts. In case of the velocity, also the maximal values present at the wave front are affected and thus a characteristic value. So it is to expect that too small values will be simulated for this quantity and as a basis for evaluation it should be used with care. A reason for the more distinct presence of numerical diffusion is that the velocity is none of the directly calculated parameters but computed a posteriori from the conserved variables of the considered system of equations. Furthermore it is to note that the production of numerical diffusion depends on utilized grid width and Courant-Friedrichs-Lewy number. Concerning transferability of simulated results to nature, it is to note that at the wave front the approach of the shallow water equations does not reflect that here exist in reality curved stream lines. Thus simulated results for this region will differ from physical processes even if they are computed without being influenced by numerical effects.

The implemented friction model by Darcy and Weisbach enables to account for friction conditions in a way close to reality.

In accordance to matters of actual research, stability of computation of especially the negative wave front on dry beds was found to be a difficulty as well as the development of roll waves in case of steep slopes and large wavelengths.

Swiss federal offices recommend simplified methods in order to appraise the reach of dam-break induced flood waves and the related risk of damages. This is of interest for the aim of classifying which kind of safety standards is to impose on dams. Reliability of such simplified methods which reduce complexity of physical coherences considerably is questioned by research groups. A comparison was drawn between results obtained with the elaborated program as reference solution and computations carried out with CASTOR, a program basing on a simplified method and ap-
plied by federal offices. It was found that considerable differences of up to one order of magnitude in the results for water height, velocity and discharge exist. Additionally, the simplified method yielded graphs of unphysical courses for the considered quantities. It is doubtful if application of this method results in realistic values and thus if basing on them decisions influencing safety of the general public should be taken.

The simulation of sediment transport due to dam-break waves is a topic of actual research. Application of the elaborated program to an experimental case yielded agreement with documented data for water heights and wave propagation. Shape and development in time of bed changes due to sediment movement correspond to observed data. Concerning computed sediment transport rates a comparison to experimental results is difficult, but in general considerably too low rates are simulated. This is ascribed to insufficient applicability of the currently available sediment transport models to dam-breaks since these models were developed for flow conditions present in rivers and estuaries.

Dam-break induced flood waves are subject of intense research. In order to continue to pursue results and approaches presented in this thesis, further work is possible in the area of numerical calculation of dam-break flood waves as well as in the areas of comparison with simplified methods and of sediment transport; thinkable measures have been pointed out.

Zusammenfassung


Die Validierung des erweiterten Programms wurde mit Hilfe analytischer, den reibungsfreien Fall betrachtender Lösungen für Dammbrüche auf horizontalen bzw. geneigten Ebenen vorgenommen sowie anhand von experimentellen Ergebnissen, die ebenfalls beide Bodengeometrien und anteilig reibungsbehäftete Ausgangssituationen berücksichtigen. Hierbei wurden gute Übereinstimmungen der simulierten Ergebnisse mit den Referenzdaten festgestellt, was Wasserhöhen, Geschwindigkeiten, Abflüsse und Forpflanzung der Wellenfront angeht.

Die räumlichen Verläufe der Parameter sind leicht durch die vom verwendeten numerischen Verfahren hervorgerufene Diffusion beeinflusst. Bei Wasserhöhe und Abfluss ist die Beeinträchtigung der Resultate als gering zu bewerten und sehr lokal auf die äußerste vordere bzw. hintere Wellenfront beschränkt. Bei der Geschwindigkeit sind Auswirkungen der numerisch bedingten Ungenauigkeit auf den an der Wellenfront auftretenden Maximalwert zu verzeichnen und somit auf einen besonders relevanten Kennwert. Daher werden hier voraussichtlich eher zu
geringe Werte berechnet und diese Größe sollte nur eingeschränkt bei Auswertungen herangezo-
gen werden. Ein Grund für das hier spürbarere Auftreten der numerischen Diffusion ist, dass
die Geschwindigkeit keine der im Gleichungssystem unmittelbar berücksichtigter Parameter ist,
sondern nachträglich aus den Erhaltungsgrößen berechnet wird. Zu beachten ist ferner, dass die
Erzeugung numerischer Diffusion in Abhängigkeit von verwendeter Diskretisierung und Courant-
Friedrichs-Lewy-Zahl steht. Bei der Übertragung der Ergebnisse auf die Natur ist hier eine Ein-
schränkung zu berücksichtigen, die aus der Verwendung der Flachwassergleichungen resultiert:
Diese basieren auf der Annahme gering gekrümmter Stromlinien, was an der positiven Wellen-
front jedoch in der Realität nicht zutrifft. Somit sind in diesem Bereich auch Ergebnisse, die
nicht durch numerische Ungenauigkeiten beeinträchtigt sind, als nur bedingt den physikalischen
Gegebenheiten entsprechend einzustufen.
Mit Hilfe des implementierten Reibungsmodells nach Darcy und Weisbach können reibungsbe-
haftete Ausgangsbedingungen realitätsnah berücksichtigt werden.
In Übereinstimmung mit der aktuellen Forschung wurde die Stabilität von Berechnungen der
Wellenfront auf trockenem Bett, insbesondere der rückwärtigen, als problematisch ermittelt;
ebenso bei der Entwicklung von sog. "Roll Waves" im Falle steller Bodengefälle und großer
Wellenlängen.
Schweizer Behörden empfehlen vereinfachte Methoden, um die Reichweite von bei Dammbrüchen
ausgelösten Flutwellen einzuschätzen und damit das Risiko, daß im Versagensfall Personen-
oder Sachschäden entstehen. Dies ist relevant, um auch kleine Stauanlagen adäquaten Bemes-
ungsvorschriften zu unterstellen. Die Zuverlässigkeit dieser die physikalischen Zusammenhänge
stark reduzierenden Verfahren wird von Forschergruppen angezweifelt. Ein Vergleich wurde
durchgeführt zwischen Ergebnissen, die mit der erweiterten CLAWPACK-Bücherei berechnet
wurden und denen eines Programms namens CASTOR, das auf einer vereinfachten Methode
beruht und von Behörden angewandt wird. Die mit CLAWPACK simulierten Ergebnisse wur-
den dabei als Referenz betrachtet. Bei den betrachteten Fällen wurden deutliche Differenzen in
den ermittelten Größen Wasserhöhe, Geschwindigkeit und Abfluss ermittelt; sie liegen bei bis zu
einer Größenordnung. Auch die mit der vereinfachten Methode berechneten Verläufe der resul-
tierenden Graphen sind zum Teil unphysikalisch. Es ist anzunehmen, ob die Anwendung dieser
 Methode zu realistischen Ergebnissen führt und somit auch, ob darauf basierend Entscheidungen
gemacht werden sollten, die die Sicherheit der Allgemeinheit beeinflussen.
Die Berechnung von Sedimenttransport infolge von Dammbruchflutwellen ist ein aktuelles For-
schungsthema. Die Anwendung des erstellten Programmes auf einen experimentellen Fall führte
zu Übereinstimmung mit dokumentierten Daten für Wasserhöhen und Fortpflanzungsgeschwin-
digkeit der Welle. Form und zeitliche Entwicklung der Sedimentverlagerungen entsprechen den
beobachteten Daten, jedoch ist hinsichtlich der transportierten Mengen ein Vergleich mit den
experimentellen Ergebnissen schwierig; tendentiell werden deutlich zu geringe Transportraten
berechnet. Dies ist auf mangelnde Anwendbarkeit der für die in Flüssen und Ästuaren herrsch-
den Strömungsbedingungen entwickelten Transportmodelle auf den Fall von Dammbrüchen zu-
rückzuführen.
Das betrachtete Gebiet der dammbruchinduzierten Flutwellen ist Gegenstand intensiver For-
schungsarbeiten. Aufbauend auf den im Rahmen dieser Arbeit erzielten Ergebnissen sind auf
allen drei betrachteten Gebieten, d.h. auf dem der numerischen Berechnung der Flutwelle, dem des Vergleichs genauer Methoden mit vereinfachten und dem des Sedimenttransports, zahlreiche weitere Arbeiten möglich und aufgezeigt worden.
A. Derivation of the Navier-Stokes Equations

In this section, the Navier-Stokes equations are derived, beginning with the conservation laws for mass and momentum.

A.1. Conservation of Mass

Reynold’s transport theorem is given by

\[
\frac{D}{Dt} \int \int \int_{V(t)} \phi \, dV = \int \int \int_{V} \frac{\partial \phi}{\partial t} \, dV + \int \int_{S} \phi u_i n_i \, dS; \tag{A.1}
\]

in this expression \( \phi \) denotes a continuous function which can be a scalar or tensor function of any order. \( S \) is the directional surface bounding \( V \), the region in space which is occupied by an enclosed part of a fluid or by a body; \( n_i \) is the normal vector.

The mass \( m \) of a bounded part of a fluid must remain constant in time:

\[
\frac{Dm}{Dt} = 0, \tag{A.2}
\]

with \( \frac{D}{Dt} \) denoting the material derivative

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i}. \tag{A.3}
\]

The mass is the sum of the mass elements \( dm \) over the set \( M \) of the material points of the body of fluid,

\[
m = \int_{(M)} dm. \tag{A.4}
\]

With respect to continuum theory, the density is considered to be a continuous function of position, so the mass can be written as the integral of the density over the region in space \( V(t) \) occupied by the body:

\[
m = \int_{(M)} dm = \int \int \int_{V(t)} \rho(\vec{x}, t) \, dV. \tag{A.5}
\]

Thus the conservation law (A.2) yields with (A.5) and Reynold’s transport theorem (A.1) the form

\[
\frac{D}{Dt} \int \int \int_{V(t)} \rho \, dV = \int \int \int_{V} \frac{\partial \rho}{\partial t} \, dV + \int \int_{S} \rho u_i n_i \, dS = 0, \tag{A.6}
\]

which gives after application of Gauss’ law to the last integral the expression

\[
\frac{D}{Dt} \int \int \int_{V(t)} \rho \, dV = \int \int \int_{V} \left[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) \right] \, dV = 0. \tag{A.7}
\]
Because this equation holds for every volume that could be occupied by the fluid or for arbitrary choice of the integration region $V$, the local or differential form of the law of conservation of mass is obtained:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho u_i) = 0.$$  \hspace{1cm} (A.8)

The material derivative yields

$$\frac{D \rho}{D t} + \rho \frac{\partial u_i}{\partial x_i} = 0.$$  \hspace{1cm} (A.9)

If the density of a material particle does not vary during its motion,

$$\frac{D \rho}{D t} = \frac{\partial \rho}{\partial t} + u_i \frac{\partial \rho}{\partial x_i} = 0 \hspace{1cm} (A.10)$$

holds. By (A.9) this is equivalent to

$$\frac{\partial u_i}{\partial x_i} = 0 \hspace{1cm} (A.11)$$

which means that the flow is volume preserving and the fluid can be viewed as incompressible.

### A.2. Conservation of Momentum

The time rate of change of the momentum of a body is equal to the total force acting on it:

$$\frac{D \vec{I}}{D t} = \vec{F},$$ \hspace{1cm} (A.12)

with $\vec{I}$ denoting the momentum, $\vec{F}$ the force acting on the body. The momentum is given by

$$\vec{I} = \iiint_{V(t)} \rho \vec{u} \, dV \hspace{1cm} (A.13)$$

as an integral over the volume of the body. The force $\vec{F}$ consists of body force $\vec{k}$, acting on a volume element of the body or fluid, and surface force $\vec{t}$ which denotes the stress or traction vector acting on a surface element at a point $\vec{x}$. Integration over the volume $V$ or surface $S$, respectively, yields

$$\vec{F} = \iiint_{V(t)} \rho \vec{k} \, dV + \iint_{S(t)} \vec{t} \, dS.$$ \hspace{1cm} (A.14)

For the balance of momentum follows

$$\frac{D}{D t} \iiint_{V(t)} \rho \vec{u} \, dV = \iiint_{V} \vec{k} \rho \, dV + \iint_{S} \vec{t} \, dS.$$ \hspace{1cm} (A.15)

Application of Leibniz’s rule and the conservation of mass, $\iiint_{V} \frac{D \rho}{D t} \, dV = 0$, yields

$$\iiint_{V(t)} \frac{D \vec{u}}{D t} \rho \, dV = \iiint_{V} \vec{k} \rho \, dV + \iint_{S} \vec{t} \, dS.$$ \hspace{1cm} (A.16)
A.3. Material Law for Newtonian Fluids and Navier-Stokes Equation

Because of assumed continuity of the integrand and the arbitrary domain of integration, this expression is equivalent to the differential form of the balance of momentum:

\[ \rho \frac{Du_i}{Dt} = \rho k_i + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (A.17) \]

where Gauss’ law was applied to the surface integral of the stress vector, using the relation \( t_i = \tau_{ij}n_j \).

Eq. (A.17) is also known as Cauchy’s first law of motion. It holds for every fluid regardless of its material properties and can be changed into a material-specific equation of motion using a constitutive equation, that is, the relationship between the stress tensor and the motion.

A.3. Material Law for Newtonian Fluids and Navier-Stokes Equation

The Cauchy-Poisson law gives a linear relationship between the stress at a material point and the deformation tensor \( e_{ij} \):

\[ \tau_{ij} = -p \delta_{ij} + \lambda^* e_{kk} \delta_{ij} + 2 \eta e_{ij}. \quad (A.18) \]

Within this material law for Newtonian fluids \( \delta_{ij} \) denotes the Kronecker delta, \( \lambda^* \) the second viscosity\(^1\), \( I \) the unity tensor and \( \eta \) the dynamic viscosity. \( e_{ij} \) is the symmetric deformation tensor

\[ e_{ij} = \frac{1}{2} \left\{ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right\}. \quad (A.19) \]

By inserting into (A.17), the Navier-Stokes equation follows:

\[ \rho \frac{Du_i}{Dt} = \rho k_i + \frac{\partial}{\partial x_i} \left\{ -p + \lambda^* \frac{\partial u_k}{\partial x_k} \right\} + \frac{\partial}{\partial x_j} \left\{ \eta \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \right\}. \quad (A.20) \]

In case of isothermal fields or neglecting the temperature dependency of \( \eta \) and \( \lambda^* \) the last term at the right-hand side of (A.20) can be written as

\[ \frac{\partial}{\partial x_j} \left\{ \eta \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] \right\} = \eta \left\{ \frac{\partial^2 u_i}{\partial x_j \partial x_i} + \frac{\partial}{\partial x_i} \left[ \frac{\partial u_k}{\partial x_k} \right] \right\}, \quad (A.21) \]

and the form

\[ \rho \frac{Du_i}{Dt} = \rho k_i - \frac{\partial p}{\partial x_i} + (\lambda^* + \eta) \frac{\partial}{\partial x_i} \left[ \frac{\partial u_k}{\partial x_k} \right] + \eta \left[ \frac{\partial^2 u_i}{\partial x_j \partial x_j} \right] \quad (A.22) \]

of the Navier-Stokes equation results. For incompressible flows where \( \frac{\partial u_k}{\partial x_k} = 0 \) holds, it follows the form of the Navier-Stokes equation in general used for water currents. It is given by

\[ \rho \frac{Du_i}{Dt} = \rho k_i - \frac{\partial p}{\partial x_i} + \eta \left[ \frac{\partial^2 u_i}{\partial x_j \partial x_j} \right] \quad (A.23) \]

\(^1\)The second viscosity considers the volumetric part of the stresses; for incompressible fluid it is identically Zero.
in symbol notation it takes the form

\[
\frac{\rho \, D\vec{u}}{Dt} = \rho \vec{F} - \nabla p + \eta \Delta \vec{u},
\]  

(A.24)

making use of the Nabla and the Laplace operator. The left-hand side can be understood as the product of the mass per volume unit of a material particle and its acceleration, the right-handed as the sum of the body force \(\rho \vec{F}\), the net pressure force per unit volume \(-\nabla p\), i.e. the difference of the pressure forces acting on the material particle\(^2\), and the net friction force per volume unit \(\eta \Delta \vec{u}\), i.e. the difference of the friction forces acting an the particle\(^3\).

\(^2\)The divergence of the pressure stress tensor \(-\nabla \cdot (\rho I)\)

\(^3\)The divergence of the friction stress tensor in incompressible current \(2\eta \nabla \cdot \mathbf{E}\)
B. Sediment Transport: Initiation of Motion

If the fluid forces acting on a sediment particle are larger than the resisting forces, particle movement will occur. The fluid forces consist of skin friction forces, acting due to viscous shear on the surface of the particles, and pressure forces. Those result from pressure differences along the surface of a particle and are split up in dragging and lifting forces $F_D$ and $F_L$. The resisting forces depend on the submerged particle weight $G$, a friction coefficient and, in case of cohesive sediment, cohesive forces. Fig. B.1 shows the forces acting on a particle. Consideration of the moments acting with respect to the point of contact yields the inequality

$$F_D (b_1 + b_2) \cos \phi + F_L b_3 \sin \phi = G b_2 \sin \phi \tag{B.1}$$

which indicates loss of stability of the particle and thus particle movement. The parameters $a_1$ to $b_3$ are depicted in fig. B.1. Eq. (B.1) gives

$$\frac{(b_1 + b_2) F_D}{(b_2 - b_3 F_L / G)} \geq G \tan \phi. \tag{B.2}$$
Assuming that the ratio of lift force and submerged particle weight is small or, more physically, that lift and drag force depend on the same variables and that therefore empirical coefficients take into account the effect of the lift force,

\[ F_D \geq \alpha_1 G \tan \phi \]  \hspace{1cm} (B.3)

is obtained from eq. (B.2). With the drag force, as a function of the drag coefficient \( C_D \) depending on the local Reynolds number, with the fluid velocity which can be expressed through the bed-shear velocity \( u_* \) and with the submerged particle weight obtainable from particle shape and the density ratio \( s = \rho_s / \rho \), eq. (B.3) yields

\[ \frac{u_*^2}{(s-1)gd} \geq \alpha_5 \tan \phi \]  \hspace{1cm} (B.4)

or, as the widely used expression used also by Shields,

\[ \theta \geq \theta_{cr} \]  \hspace{1cm} (B.5)

Within this equation, the mobility parameter \( \theta \) and the critical Shields parameter \( \theta_{cr} \) are defined as

\[ \theta = \frac{u_*^2}{(s-1)gd} = \frac{\tau_b}{(\rho_s - \rho)gd} \]  \hspace{1cm} (B.6)

\[ \theta_{cr} = \alpha_5 \tan \phi = \frac{4\alpha_1}{3\alpha_2^2 C_D} \tan \phi \]  \hspace{1cm} (B.7)

with \( \tau_b \) as the bed shear stress and \( \alpha_1, \alpha_2 \) and \( \alpha_5 \) as parameters depending on the local Reynolds number \( Re_* \), \( \theta_{cr} = \theta_{cr}(Re_*) \), and defined by Van Rijn in [67, pp.4.1-4.3]. The local or grain-related Reynolds number near the bed is defined through

\[ Re_* = u_* d / \nu, \]  \hspace{1cm} (B.8)

where according to experimental results of Shields the characteristic particle diameter \( d \) is the median diameter \( d = d_{50} \). Fig. B.2 shows the Shields curve depending on \( \theta_{cr} \) and \( Re_* \).

The influence of longitudinal bed slope on the initiation of motion can be accounted for through a factor \( k_\beta \), introduced by Schoklitsch. For an inclined bed as depicted in fig. B.3, eq. (B.3) reads for \( \alpha_1 = 1 \) and the case of equality of critical fluid force \( F_{D,cr} \) and stabilizing weight force

\[ F_{D,cr} + G \sin \beta = G \cos \beta \tan \phi \]  \hspace{1cm} (B.9)

For a horizontal bed, the critical fluid force \( F_{D,cr,0} \) is obtained through eq. (B.3) with again \( \alpha_1 = 1 \) as

\[ F_{D,cr,0} = G \tan \phi. \]  \hspace{1cm} (B.10)

The ratio of both critical fluid forces yields

\[ \frac{F_{D,cr}}{F_{D,cr,0}} = \frac{G \cos \beta \tan \phi - G \sin \beta}{G \tan \phi} = \frac{\sin(\phi - \beta)}{\sin \phi}, \]  \hspace{1cm} (B.11)
Figure B.2.: Shields curve; [67]

Figure B.3.: Forces acting on a particle; [67]
relating the critical fluid force on a sloping bed to the critical fluid force on a horizontal bed by

$$F_{D,cr} = k_\beta F_{D,cr,0}. \quad (B.12)$$

As the critical fluid force or bed shear stress should be increased for upsloping flow and decreased for downsloping flow, the Schoklitsch correction factor is defined as

$$k_\beta = \begin{cases} \frac{\sin(\phi + \beta)}{\sin(\phi)} & \text{for upsloping flow, } k > 1, \\ \frac{\sin(\phi - \beta)}{\sin(\phi)} & \text{for downsloping flow, } k < 1. \end{cases} \quad (B.13)$$

In terms of the bed shear stress, equation (B.12) reads

$$\tau_{b,cr} = k_\beta \tau_{b,cr,0}. \quad (B.14)$$

Similarly, an expression for transverse slope with a correction factor $k_\gamma$ is obtained from consideration of the force diagram shown in fig. B.4 and the force equilibrium

$$\sqrt{F_{D,cr}^2 + G \sin^2 \gamma} = G \cos \gamma \tan \phi. \quad (B.15)$$

The correction factor, introduced by Leiner, reads

$$k_\gamma = \cos \gamma \left(1 - \frac{\tan^2 \gamma}{\tan^2 \phi}\right)^{0.5} \quad (B.16)$$

for the modification

$$F_{D,cr} = k_\gamma F_{D,cr,0} \quad (B.17)$$

or

$$\tau_{b,cr} = k_\gamma \tau_{b,cr,0} \quad (B.18)$$

of the critical fluid force and the critical bed shear stress, respectively.
Bibliography


