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Computing granular avalanches across irregular topography

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ABSTRACT: The Savage-Hutter (SH) equations for dry granular flows are a system of hyperbolic balance laws, which is based on a Coulomb friction approach for the description of internal failure and basal sliding and determines the time-dependent behavior of depth and depth-integrated velocity components in a terrain following coordinate system. In this paper we present results derived with a new numerical model, operating in the finite-volume context with a shock-capturing wave-propagation method. The numerical model is applied on a laboratory test problem, a dry granular avalanche down an irregular topography and compared against the corresponding laboratory experiments.

1 INTRODUCTION

Debris flows, landslides, mud-flows and hyper-concentrated flows, etc. occur when masses of poorly sorted sediment, agitated and saturated with water, flow down slopes in response to gravitational acceleration. All gravity-driven mass movements can occur with little warning. The capricious timing and magnitude of these gravity-driven mass flow events hamper collection of detailed data. Scientific understanding has thus been gleaned mostly from qualitative field observations and highly idealized first generation experiments and models.

Here we are focused in a first step on a sort of hybrid modeling, the results of a numerical experiment, a granular avalanche down an irregular laboratory topography are shown and compared against the corresponding laboratory experiments.

2 MATHEMATICAL MODEL

Many of these gravity-driven flow phenomena can be characterized as shallow flows. One popular model family describing the time-dependent behavior of gravity-driven multiphase free-surface flows (e.g. snow avalanches, landslides, rock falls and debris flows) is based on the Savage-Hutter theory (Savage & Hutter 1989; Gray et al. 1998). It assumes an incompressible shallow flow behavior and that the flowing mass behaves as a Mohr-Coulomb plastic material at yield. The basal shear stress is therefore equal to the normal basal pressure multiplied by a friction coefficient $\tan \phi_{bed}$, with the basal friction angle ϕ_{bed}. Hereby *bed* stands for bed level.

In conservation-law form these equations in 2-dimensional vector notation are presented below. They describe the flow at time $t \geq 0$ at $x, y \in \mathbb{R}$ and read

$$\partial_t \vec{q} + \partial_x \vec{f}(\vec{q}) + \partial_y \vec{g}(\vec{q}) = \vec{S}(\vec{q}) \,, \tag{1}$$

with

$$\vec{q} := \vec{q}(x, y, t) := (h,\ hu,\ hv)^T \,, \tag{2}$$

the flux vectors

$$\vec{f}(\vec{q}) := \begin{pmatrix} hu \\ hu^2 + \frac{1}{2} k_{a/p} g_z h^2 \\ \rho uv \end{pmatrix} ;$$

$$\vec{g}(\vec{q}) := \begin{pmatrix} \rho v \\ \rho vu \\ \rho v^2 + \frac{1}{2} k_{a/p} g_z h^2 \end{pmatrix} ,$$

and the source term vector, representing the net driving force

$$\vec{S}(\vec{q}) := \begin{pmatrix} 0 \\ s_x \\ s_y \end{pmatrix} .$$

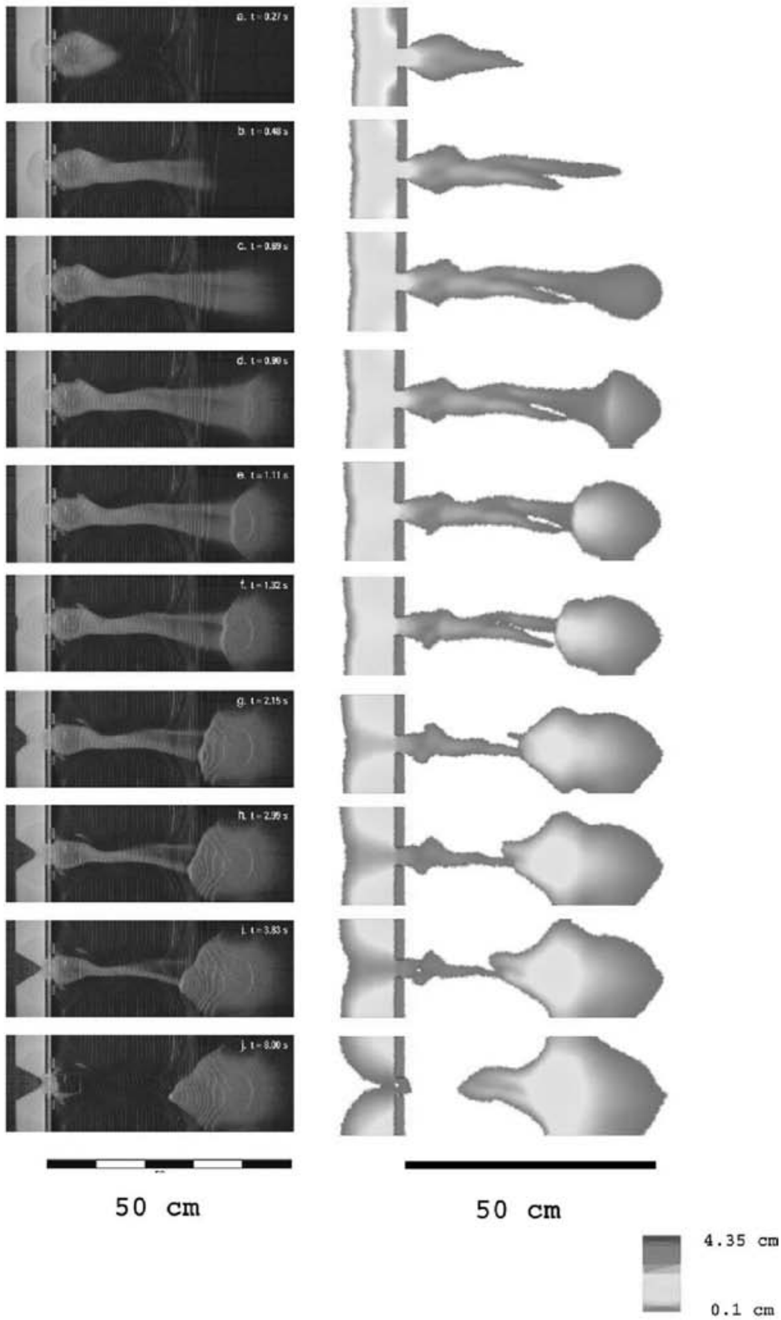

Figure 1. Flow height results of the granular avalanche down an irregular laboratory topography as isoline representation. Left: Laboratory experiments (Courtesy Iverson et al. (2004)). Contours depict 0.5 mm isopachs of flow thickness normal to the bed. Right: Numerical experiments. Color coding represents the flow height range between the minimum (0.0 cm) and the maximum flow height of 4.35 cm corresponding the material initial height.

In the expression for $\vec{S}(\vec{q})$ the terms represent the source contributions of the Cauchy stress tensor s_x, s_y and are given by:

$$s_x = g_x h - \frac{u}{|\vec{v}|}\left(g_z + u^2 \frac{\partial \Theta_x}{\partial x}\right) h \tan\phi_{\text{bed}}$$

$$- \operatorname{sgn}\left(\frac{\partial u}{\partial y}\right) h k_{a/p} \frac{\partial}{\partial y}\left[g_z h \sin\phi_{int}\right].$$

and

$$s_y = g_y h - \frac{v}{|\vec{v}|}\left(g_z + v^2 \frac{\partial \Theta_y}{\partial y}\right) h \tan\phi_{bed}$$

$$- \operatorname{sgn}\left(\frac{\partial v}{\partial x}\right) h k_{a/p} \frac{\partial}{\partial x}\left[g_z h \sin\phi_{int}\right]. \qquad (3)$$

Here $k_{a/p}$ is the earth pressure coefficient, representing the ratio between the normal stress in the down- or cross-slope direction and the vertical normal stress

$$k_{a/p} = \begin{cases} 2\frac{1-\left[1-\cos^2\phi_{int}(1+\tan^2\phi_{bed})\right]^{1/2}}{\cos^2\phi_{int}} - 1 & : \\ \qquad \left[\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) < 0\right] \\ 2\frac{1+\left[1-\cos^2\phi_{int}(1+\tan^2\phi_{bed})\right]^{1/2}}{\cos^2\phi_{int}} - 1 & : \\ \qquad \left[\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) > 0\right]. \end{cases}$$

The subscript a indicates an active stress state corresponding to a dilatation $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) < 0$ of the material and the subscript p a passive stress state associated with a compression $\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) > 0$.

In this notation h is the flow depth normal to the local bed surface, $\vec{v} = (u, v, 0)^T$ the vector of the depth-averaged velocity, $\vec{g} = (g_x, g_y, g_z)^T$ the vector of the gravitational acceleration, ϕ_{int}, ϕ_{bed} the internal and the bed friction angle, v_f the fluid volume fraction, z_{bed} the bottom height function and $\vec{\Theta} = (\Theta_x, \Theta_y, 0)^T$ the vector of the angle of the local bed slope.

3 NUMERICAL EXPERIMENT

The numerical model solving the system of equations Eqns. (1) works in a finite volume context, on unstructured triangular grids and realizes a higher order Riemann solver based wave-propagation method developed by Vollmöller (2004b). This scheme is necessary for the accurate resolution of sharp fronts and superior for the incorporation of complicated source terms (Vollmöller 2004a). The described numerical model is applied to a granular avalanche down an irregular laboratory topography and compared against the corresponding laboratory experiments done by Iverson et al. (2004), which serve as reference solutions.

3.1 *Granular avalanche down irregular topography*

The computational domain in the irregular topography experiment has the dimensions $x \in [0, 1] \times y \in [-0.1, 0.1]$ (m^2) and consists of an inclined region ($0 < x < 0.1$ m), a horizontal runout ($x > 0.39$ m) and an irregular basal surface for $0 < x < 0.39$ m. The head of the flume contains a head gate of the width 4 cm for a sudden release. The dry granular mixture has a total volume of 308 cm^3 and an initial flow height of the 4.35 cm behind the gate. The mass starts in all experiments from rest, so the initial condition for the velocity is $\vec{v} = (0, 0, 0)^T$. The boundaries are closed at the sides parallel and open at the sides rectangular to the downslope or flow direction. The computations are carried out on a static numerical grid with $2 \cdot 10^4$ grid cells.

Fig. 1 shows the computational results of the experiment compared to the laboratory experiments for five timeslices. The agreement between both is good, which stresses the fact that the numerical scheme and the physical model in combination represent the natural condition in a reasonable manner.

4 CONCLUSION

In this paper a numerical model for the description of the dynamical behavior of dry granular avalanches was developed. It basis on the Savage-Hutter equations, a system of hyperbolic balance laws, taking the net driving force of the flow via source terms, representing the constitutive material relations, into account. These balance laws describe the time-dependent flow behavior of a dry granular flow over complex three-dimensional basal topographies. The numerical model is applied on a dry granular avalanche down an irregular topography. The results between the numerical and the corresponding laboratory experiment are in good agreement and encourage the application of this numerical model on real large scale events.

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