

# Visco-plastic fluids: from Theory to Application

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Fig. 1. The participants enjoying the sunny conditions of Tecino.

Viscoplastic materials are fluids that exhibit a yield stress: below a certain critical threshold in the imposed stresses, there is no deformation and the material behaves like a rigid solid, but when that yield value is exceeded, the material flows like a fluid. Practically, such flow behavior appears in many situations, including slurries and suspensions, certain polymer solutions, crystallizing lavas, muds and clays, heavy oils, avalanches, cosmetic creams, hair gel, liquid chocolate, and some pastes. Consequently, the theory of the fluid mechanics of such materials has applications in wide array of different fields, ranging from the oil, gas and chemical industries, to food processing and to geophysical fluid dynamics. The most commonly studied such fluid is the Bingham fluid, which is often uppermost in the minds of the scientists when they think of non-Newtonian fluids, although in fact this model is usually a significant simplification of the true rheological behavior.

From the 14th to 18th October 2007, a workshop gathering 49 participants from 13 countries was held in Monte Verità, a marvelous place in the Italian-speaking part of Switzerland, with spectacular scenery on lake Major and the city of Locarno. The meeting was the follow-on of an earlier workshop organized at Banff (Canada) in 2005. This was the opportunity to have a closer look at recent achievements in this hectic field and try to collectively think of future developments. We also paid homage to the Greek and Cypriot communities for their enduring efforts in viscoplastic fluid mechanics. During an enjoyable banquet, Evan Mitsoulis was awarded the Bingham Fluid Medal (not to be confused with a more auspicious prize) for his research on computational aspects of viscoplasticity while John Tsamopoulos was rewarded with a prize for successfully computing the shape of the yield surfaces around bubbles in Bingham fluids. The next workshop will hold in Cyprus in October 2009.

Part of the objective of this workshop was to promote exchanges between scientists from different origins, which made it possible to shed light on a number of topical issues in different ways. Here is a rapid summary of the different presentations and discussions.

The nature of yield stress is a longstanding problem, which has received considerable attention over the last 20 years. The emblematic question may be the suitability of the Herschel-Bulkley equation to represent the true rheological behavior over a sufficiently wide range of shear rates. The tentative answer provided by Coussot, Bonn, and Molder is that while the notion of yield stress makes sense experimentally, there is also a critical shear rate below which no steady state is observed. This critical rate not only contrasts with the predictions of the Herschel-Bulkley model, but also gives rise to flow instabilities (e.g., shear banding) due to viscosity bifurcation. Understanding the physical origin of yielding on the particle scale is of paramount importance to appreciate empirical bulk-scale idealizations such as the Bingham

model. Van Damme showed how adding adsorbing or non-adsorbing polymers in a colloidal suspension changes the yield-stress and bulk-viscosity values, which emphasizes the role played by particle contacts (lubricated or direct) in yielding; a similar behavior was reported by Bardou with debris-flow samples. Nicot summarized the various definitions used in geomechanics to define the yield surface on the bulk and particle scales; a key result is the existence of different modes of failure for a dry granular material. Coussot and Alexandrou supplemented the Herschel-Bulkley constitutive equation with a scalar kinetic equation reflecting the changes in particle arrangement (breakdown/builtup of links between particles). A more formal continuum-mechanics approach was proposed by Goddard, who showed that a host of phenomena can be described using a generalized viscosity coefficient (fourth-rank viscosity tensor).

From this confrontation between experimental observations, phenomenological laws and theoretical views of particulate media in geomechanics and fluid mechanics arose a number of questions that deserve mention. Are there hidden issues with the shape of the yield surface in principal stress space (i.e. non-von-Mises yield laws) that we have not appreciated so far? Are plasticity theory's flow laws better than the current rate-dependent terms of viscoplastic constitutive models? Should we pay more attention to issues of symmetry/invariance and violation of physical principles (e.g., the second law of thermodynamics)? Is there interest for theoreticians to explore and understand more of the underlying physics, e.g. continue the search for microstructural theories to build macroscopic flow models?

The notions of yield surface and post-yielding behavior has evolved differently in geomechanics and fluid mechanics. As illustrated in Chevalier's experiments and Pastor's talk, yielding of saturated granular materials may be the consequence of a pore-pressure increase and particle fluidization. This highlights the two-phase behavior of apparently single-phase bulk materials. Another non-trivial question, notably addressed in Wilson's squeezing flow experiments, is related to the influence of the mean stress (first invariant) on yield stress: once the material has yielded, does the shear stress still vary linearly with the normal stress? Goddard's theoretical analysis and Pouliquen's phenomenological interpretation provided evidence of both shear-rate and normal-stress dependence of the shear stress in dry and saturated granular materials. The different talks underlined the gap between classical single-phase viscoplastic models and plastic multi-phase Coulomb-like theories. To date, there is little connection. Should we try to investigate Bingham-type models with pore-pressure-dependent yield stresses, and so forth? Is liquifaction something we should be considering?

Whereas earlier experiments focused on viscometric flows in rheometers, most recent experimental investigations have been done using more complex geometries. In parallel, image processing has been extensively used to take measure-

ments and visualize what is going on inside the material. Hogg reviewed recent theoretical results for the dam-break problem, which can be seen as the prototypical time-dependent problem, while Cochara showed his experimental results for this geometry. As shown by Balmforth, another interesting time-dependent problem is the Stokes problem, where a viscoplastic material is confined between two parallel walls, one being in oscillating motion, the other being fixed. Chambon used a conveyor belt to investigate the inner structure of a finite volume of Carbopol in a steady regime. Cheddadi, de Bruyn and Mitsoulis showed how flows past an obstacle or a falling sphere can be effectively investigated experimentally and numerically.

Classical geometries such as the Couette cell still attract much attention. Compared to other rheometer geometries, wide-gap Couette cells are well suited to particle suspensions, but flow-curve inference may be more delicate. Wiederseiner used transparent buoyant-particle suspensions and particle-tracking velocimetry to estimate the local flow curve. Naccache compared different boundaries (grooved surface, vane) to minimize slip effects at the wall. There was little discussion about experimental techniques for measuring the flow properties inside the material; while magnetic resonance imaging is now arrived at a mature stage, it remains an expensive and heavy technique. We are on the verge of important developments in image processing that should make flow visualization easier and cheaper, but probing what occurs in opaque materials still offers substantial difficulties.

Since Glowinski's pioneering work, a great deal of work has been accomplished to provide robust and accurate algorithms able to cope with viscoplastic behavior. As reviewed by Huilgol and exemplified in a number of applications shown by Picasso and Muravleva, variational methods are in common use. In particular, regularization techniques such as augmented Lagrangian and related methods have demonstrated their efficiency. Saramito provided another perspective by showing how an adaptive meshing strategy can be used to increase the order of a numerical scheme.

Since the Banff meeting, where we suggested using Carbopol 940 as a suitable viscoplastic fluid, evidence has accumulated that Carbopol is far from being an ideal Herschel-Bulkley fluid; in particular, elasticity cannot be neglected at low shear stress. This has led to a substantial renewal of interest in elasto/viscoplasticity and plastic compressibility. Recent models and applications were presented by Saramito, Forterre, Georgiou, and de Souza Mendes. In parallel, the quest for a genuinely Herschel-Bulkley fluid is launched again. Graner's experiments revealed the value of two-dimensional foams to investigate plastic flow problems, but elasticity still plays a role in the bulk-stress generation. From the presentations, it appeared that Carbopol requires relatively long mixing times in order to achieve reproducible results. There seems to be some need for guidelines in preparing Carbopol (and other viscoplastic candidates),

a database listing standard Herschel-Bulkley fits as functions of concentration and  $pH$ , and more sophisticated viscoplastic models incorporating elastic effects and slip.

A number of particular applications were also presented. As shown by Moyers, Nouar and Rust, interfacial, thermal, and/or convective instabilities in industrial ducts or in magmatic chambers lead to complex problems in stability analysis. Fester and Slatter explained why determining energy losses in turbulent/laminar viscoplastic flows is of great importance for pipeline design and operation. Geomaterials involved in natural gravity-driven flows have often been modeled as plastic fluids. Bartelt reviewed some key questions in the rheology of snow and recast rheological questions within a thermodynamics framework by highlighting the role of energy and mass transfers in snow avalanches. As shown by Bonnoit, McElwaine, and Kowalski, there is still a host of questions (e.g., segregation in granular flows, behavior of multicomponent fluids) that have received partial answers so far. Today, in the realm of geophysical flows, it is still unclear how to forge a symbiotic relation between engineers who use crude shallow-layer theories coupled with Bingham-like drag parameterizations (amongst others) and physicists who are developing more physically-based theories on the basis of laboratory data. Can we help to better constrain those former models with the latter type of approaches? Turbulence in viscoplastic seems to be an open field for research: when does transition occur, and how are friction factors affected by viscoplasticity? What is the phenomenology of viscoplastic turbulence? Can it be characterized by relatively simple dimensional scaling theories and ideas about the effect of yield stress on the energy containing scales, the inertial range and the dissipation scales?

The meeting was scientifically rewarding and enjoyable, which encouraged the organization of subsequent workshops with the same format. Monte Verità was a striking venue for a focused workshop of this nature. We are very grateful to Centro Stefano Franscini for their financial and technical support.